# Løsning: Oppgavesett 9

**1.** a) $E(\bar{x}) = E\left(\frac{1}{n}(x_1 + \dots + x_n)\right)$

$= \frac{1}{n}\left(E(x_1) + \dots + E(x_n)\right)$

$= \frac{1}{n}(\mu + \mu + \dots + \mu) = \frac{1}{n} \cdot n\mu = \underline{\underline{\mu}}$

b) $Var(\bar{x}) = Var\left(\frac{1}{n}(x_1 + \dots + x_n)\right)$

$= \frac{1}{n^2}\left(Var(x_1) + Var(x_2) + \dots + Var(x_n)\right)$

$x_1, \dots, x_n$ uavhengige

$= \frac{1}{n^2} \cdot n \cdot \sigma^2 = \underline{\underline{\sigma^2/n}}$

$SE(\bar{x}) = \sqrt{\sigma^2/n} = \underline{\underline{\sigma/\sqrt{n}}}$

c) <u>Punktestimat:</u>

kalkulator

$\bar{x} = \frac{x_1 + \dots + x_{25}}{25} = \frac{104 + \dots + 97}{25} = \underline{\underline{98.04}}$

d) $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}\sum\left(x_i^2 - 2\bar{x}X_i + \bar{x}^2\right)$

$= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} X_i + n\bar{X}^2\right]$

$= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - 2\bar{x} \cdot n\bar{X} + n\bar{X}^2\right]$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right]$$

$$E(S^2) = E\left(\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right]\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} E(x_i^2) - n \cdot E(\bar{x}^2)\right)$$

$$= \frac{1}{n-1}\left(n(\sigma^2+\mu^2) - n\left(\frac{\sigma^2}{n}+\mu^2\right)\right)$$

$$= \frac{1}{n-1}\left(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2\right)$$

$$= \frac{1}{n-1}\left((n-1)\sigma^2\right) = \underline{\underline{\sigma^2}}$$

$$Var(x_i) = E(x_i^2) - E(x_i)^2$$

$$\Downarrow$$

$$E(x_i^2) = Var(x_i) + E(x_i)^2 = \sigma^2 + \mu^2$$

$$Var(\bar{x}) = E(\bar{x}^2) - E(\bar{x})^2$$

$$\Downarrow$$

$$E(\bar{x}^2) = Var(\bar{x}) + E(\bar{x})^2 = \frac{\sigma^2}{n} + \mu^2$$

e) $$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$= \frac{1}{24}\left((104-98.04)^2 + \dots + (97-98.04)^2\right) = \underline{\underline{133.7}}$$

kalkulator

## 2.

$$E(\hat{p}) = E(X/n) = \frac{1}{n}E(X) = \frac{1}{n}\cdot np = \underline{\underline{p}}$$

$$Var(\hat{p}) = Var(X/n) = \frac{1}{n^2}\cdot np(1-p)$$

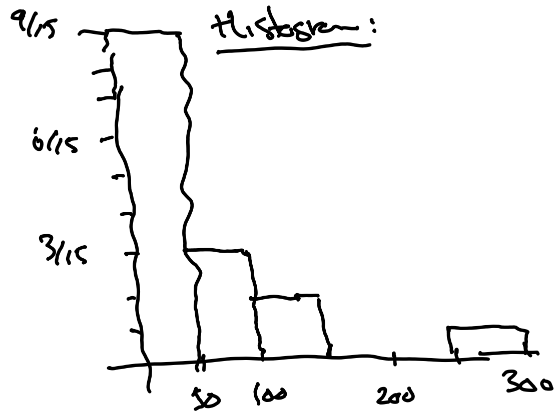
$$= \underline{\underline{\frac{p(1-p)}{n}}}$$

## 3.

a) Frekvenstabell:

| | | |
|---|---|---|
| 1-50 | 9 | 9/15 |
| 51-100 | 3 | 3/15 |
| 101-150 | 2 | 2/15 |
| 151-200 | 0 | 0 |
| 201-250 | 0 | 0 |
| 251-300 | 1 | 1/15 |

Histogram:




representerer sannsynlighetsfordelingen

b) $\bar{x} = \frac{1}{n}(x_1 + \dots + x_n) = \frac{23 + \dots + 49}{15} \approx \underline{\underline{63.7}}$ (kalk.)

modus: $\underline{\underline{34}}$ (forekommer flest ganger)

median: 2 10 11 12 15 23 34 34 49 72 78 81 101 144 290

$\frac{15+1}{2} = 8 \Rightarrow$ median $= \underline{\underline{34}}$

gjennomsnitt, modus, median er sentralmål, skal representere en typisk verdi

gjennomsnitt er mye høyere enn modus/median fordi enkelte målinger er svært mye høyere enn de andre målingene i datasettet

c) Kvartil:

$\frac{15+1}{4} = 4$ Nedre kvartil: $\underline{\underline{12}}$

$\frac{15+1}{4} \cdot 3 = 12$ Øvre kvartil: $\underline{\underline{81}}$

(måling nr 4 og 12, i stigende rekkefølge)

Boksplott:


d) Variasjonsbredde: $290 - 2 = \underline{288}$

Kvartilbredde: $81 - 12 = \underline{69}$

kalk.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{(23-63.7)^2 + \ldots + (49-63.7)^2}{14} \simeq 5555$$

Standardavvik: $s \simeq \sqrt{5555} \simeq \underline{\underline{74.5}}$

Variasjonskoeff: $\frac{s}{\bar{x}} = \frac{74.5}{63,7} \simeq 1.17 = \underline{\underline{117\%}}$

4.

a) Både $X_1$ og $X_2$ er gitt ved $\bar{X}$ (for forskjellige utvalg). Siden

$$E(\bar{X}) = \mu \quad \text{er forventningsrett}$$

er $X_1, X_2$ forventningsrette.

b)
$$Var(X_1) = \frac{\sigma^2}{n} = \frac{\sigma^2}{10} \Rightarrow SE(X_1) = \underline{\underline{\sigma/\sqrt{10}}}$$

$$Var(X_2) = \frac{\sigma^2}{n} = \frac{\sigma^2}{40} \Rightarrow SE(X_2) = \underline{\underline{\sigma/\sqrt{40}}} = \frac{\sigma}{2\cdot\sqrt{10}}$$

antar $X_1, X_2$ uavh. siden uavh. trekninger

c)
$$X_3 = 0.5X_1 + 0.5X_2$$

$$Var(X_3) = 0.5^2 \cdot Var(X_1) + 0.5^2 \cdot Var(X_2)$$

$$= \frac{1}{4}\cdot\frac{\sigma^2}{10} + \frac{1}{4}\cdot\frac{\sigma^2}{40} = \frac{4\sigma^2}{160} + \frac{\sigma^2}{160}$$

$$= \frac{5\sigma^2}{160} \Rightarrow SE(X_3) = \underline{\underline{\sqrt{\frac{5}{160}}\,\sigma}}$$

$$X_4 = 0.2X_1 + 0.8X_2$$

$$Var(X_4) = 0.2^2 \cdot \frac{\sigma^2}{10} + 0.8^2 \cdot \frac{\sigma^2}{40} = \frac{1}{25}\frac{\sigma^2}{10} + \frac{16}{25}\frac{\sigma^2}{40}$$

$$= \frac{\sigma^2}{250} + \frac{4\sigma^2}{250} = \frac{5\sigma^2}{250} = \frac{1}{50}\sigma^2 \Rightarrow SE(X_4) = \underline{\underline{\sigma/\sqrt{50}}}$$

5. Se løsning for Eksamen MET1190, 11/2018, oppg. 1a-d