Løsning: Oppgaveark 8

1.

i	x_i	y_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$y_i-\bar{y}$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
1	75	1050	-22	484	-60	3600	1320
2	145	1100	48	2304	-10	100	-480
3	55	1200	-42	1764	90	8100	-3780
4	88	1120	-9	81	60	3600	-540
5	122	1030	25	625	-80	6400	-2000
	485	5550		5258		21.800	-5480

a) $\bar{x} = \frac{485}{5} = \underline{\underline{97}}$

b) $s_x^2 = \frac{5258}{4} = \underline{\underline{1314,5}}$ $\Rightarrow$ c) $s_x \approx \underline{\underline{56,3}}$

d) $\bar{y} = \frac{5550}{5} = \underline{\underline{1110}}$

e) $s_y^2 = \frac{21800}{4} = \underline{\underline{5450}}$ $\Rightarrow$ f) $s_y \approx \underline{\underline{73.8}}$

g) $s_{xy} = \frac{-5480}{4} = \underline{\underline{-1370}}$ $\Rightarrow$

h) $r = \frac{s_{xy}}{s_x \cdot s_y} \approx \frac{-1370}{56.3 \cdot 73.8} \approx \underline{\underline{-0.512}}$

2. Samme svar som ovenfor.

s_x^2: [S_x] [x^2] s_y^2 = [S_y] [x^2]

s_{xy}: [r] * [S_x] * [S_y]

3.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{75} \sum_{i=1}^{75} x_i$$

$$\Rightarrow \sum_{i=1}^{n} x_i = 75 \cdot \bar{x} = 75 \cdot 12 = \underline{\underline{900}}$$

4.

$$S_x^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 \geq 0$$

sidn $\frac{1}{n-1} = \frac{1}{134} > 0$

og $(x_i - \bar{x})^2 \geq 0$ for alle i

$$S_x^2 = 0 \Leftrightarrow x_i - \bar{x} = 0 \text{ for alle } i$$

$x_i = \bar{x}$ — " —

$\underline{x_1 = x_2 = \ldots = x_{135} = 23}$

5.

a) $1+2+3+4+5 = \underline{\underline{\sum_{i=1}^{5} i}}$

b) $1+3+5+7+9+11 = \underline{\underline{\sum_{i=1}^{6} (2i-1)}}$

c) $1+3+5+\ldots+101 = \underline{\underline{\sum_{i=1}^{51} (2i-1)}}$

d) $x_1 + \ldots + x_{21} = \underline{\underline{\sum_{i=1}^{21} x_i}}$

e) $x_1^2 + \dots + x_{13}^2 = \sum_{i=1}^{13} x_i^2$

f) $(x_1-\bar{x})^2 + \dots + (x_{11}-\bar{x})^2 = \sum_{i=1}^{11} (x_i-\bar{x})^2$

g) $2+4+6+\dots+200 = \sum_{i=1}^{100} (2i)$

h) $101 + \dots + 199 = \sum_{i=51}^{100} (2i-1) = \sum_{i=1}^{50} (2i+99)$

6.

a) $\sum_{i=1}^{4} (2i+1) = 3+5+7+9 = 24$

b) $\sum_{i=1}^{n} (2i+1) = 3+5+7+\dots+(2n+1)$

$= \frac{3+2n+1}{2} \cdot n = (n+2)n$

Formel for aritmetisk rekke:

$S_n = a_1 + a_2 + \dots + a_n = \frac{a_1 + a_n}{2} \cdot n$

når $a_i - a_{i-1} = d$ er konstant

c) $\sum_{i=1}^{4} i(2i-1) = 1\cdot 1 + 2\cdot 3 + 3\cdot 5 + 4\cdot 7$

$= 1 + 6 + 15 + 28 = 50$

d) $\sum_{i=1}^{4} (2i-1)^2 = 1^2 + 3^2 + 5^2 + 7^2$

$= 1 + 9 + 25 + 49 = \underline{\underline{84}}$

e) $\sum_{i=1}^{4} \left[i(2i-1)^2 - 2 \right] = (1 \cdot 1^2 - 2) + (2 \cdot 3^2 - 2) + (3 \cdot 5^2 - 2)$

$+ (4 \cdot 7^2 - 2) = 1 \cdot 1 + 2 \cdot 9 + 3 \cdot 25 + 4 \cdot 49 - 4 \cdot 2$

$= 1 + 18 + 75 + 196 - 8 = \underline{\underline{282}}$

7.

a) $\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i = \frac{121}{10} = \underline{\underline{12.1}}$

b) $\sum_{i=1}^{10} (x_i + 3) = \sum_{i=1}^{10} x_i + 10 \cdot 3 = 121 + 30 = \underline{\underline{151}}$

c) $\sum_{i=1}^{10} (2x_i - 1) = 2 \cdot \sum x_i - 10 \cdot 1 = 2 \cdot 121 - 10$

$= 242 - 10 = \underline{\underline{232}}$

d) $\sum_{i=1}^{10} (x_i - 1)^2 = \sum x_i^2 - \sum 2x_i + \sum 1$

$= 1777 - 2 \cdot 121 + 10 \cdot 1 = \underline{\underline{1545}}$

e) $\sum_{i=1}^{10} \left((2x_i - 1)^2 + 4 \right) = \sum 4x_i^2 - \sum 4x_i + 10 \cdot 1 + 10 \cdot 4$

$= 4 \cdot 1777 - 4 \cdot 121 + 50 = \underline{\underline{6674}}$

f) $s_x^2 = \frac{1}{9} \sum (x_i - \bar{x})^2 = \frac{1}{9}\left(\sum x_i^2 - \sum 2\bar{x} x_i + \sum \bar{x}^2 \right)$

$= \frac{1}{9}\left(1777 - 2 \cdot 12.1 \cdot 121 + 10 \cdot 12.1^2\right)$

$\simeq \underline{34.77} \Rightarrow s_x = \sqrt{s_x^2} \simeq \underline{\underline{5.90}}$

8.

a) $\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - n \cdot \bar{x} = \sum_{i=1}^{n} x_i - n\left(\frac{1}{n} \sum_{i=1}^{n} x_i\right)$

$= 0$

$\Rightarrow \underline{\underline{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x}) = 0}}$

b) $\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} \left(x_i^2 - 2\bar{x} x_i + \bar{x}^2\right)$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x} \sum_{i=1}^{n} x_i + n\bar{x}^2$

$= \sum x_i^2 - 2\bar{x} \cdot n \cdot \bar{x} + n\bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

$\Rightarrow \underline{\underline{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}}$

c) $\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \sum_{i=1}^{n}(x_iy_i - \bar{x}y_i - \bar{y}x_i + \bar{x}\bar{y})$

$= \sum_{i=1}^{n} x_iy_i - \bar{x}(n\bar{y}) - \bar{y}(n\bar{x}) + n\bar{x}\bar{y}$

$= \sum_{i=1}^{n}(x_iy_i) - n\bar{x}\bar{y}$

$\Rightarrow \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{n-1}\left(\sum_{i=1}^{n}(x_iy_i) - n\bar{x}\bar{y}\right)$

9. Fra 8b): $\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}x_i^2 - n\bar{x}^2 \geq 0$

fordi VS er sum av kvadrater

$\Downarrow$

$\sum_{i=1}^{n}x_i^2 - n\bar{x}^2 \geq 0$

$\sum_{i=1}^{n}x_i^2 \geq n\bar{x}^2 = n\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$= \frac{n}{n^2}\left(\sum_{i=1}^{n}x_i\right)^2 = \frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)^2$

Det følger at

$$\sum_{i=1}^{n}x_i^2 \geq \frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)^2$$

for alle data $x_1 \dots x_n$

Hvis $x_1, \dots, x_{100}$ er slik at $\sum_{i=1}^{100} x_i = 1156$,

så er

$$\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 = \frac{1}{100}\cdot(1156)^2$$

$$= 13.363,36$$

$\Downarrow$

$$\sum x_i^2 \geq 13.363,36$$

<u>Konklusjon:</u> $\sum_{i=1}^{n} x_i^2 = 13.359$

er <u>ikke mulig</u>, pga

$$\sum_{i=1}^{n} x_i^2 \geq \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$$