Løsning: Oppgavesett 4

<u>1.</u> $X \sim \text{Binom}(n=3,\ p=1/3)$

siden $p(\{5,6\}) = 2/6 = 1/3$

a) $p(x=1) = \binom{3}{1}(1/3)^1(2/3)^2 = 3 \cdot 1/3 \cdot 4/9 = \underline{\underline{4/9}}$

b) $p(X>1) = 1 - p(X \leq 1) = 1 - p(x=0) - p(x=1)$

$= 1 - \binom{3}{0} \cdot (1/3)^0 \cdot (2/3)^3 - 4/9$

$= 1 - \frac{8}{27} - \frac{4}{9} = \frac{27 - 8 - 4 \cdot 3}{27} = \underline{\underline{7/27}}$

c) $\mu = E(x) = np = 3 \cdot 1/3 = \underline{\underline{1}}$

d) $Var(x) = npq = 3 \cdot 1/3 \cdot 2/3 = 2/3$

$\Rightarrow \sigma = \underline{\underline{\sqrt{2/3}}} \simeq \underline{\underline{0.82}}$

2. R = antall riktige svar

$$= \underbrace{5}_{\text{sikre}} + \underbrace{S}_{\text{antall riktige blant de ti der du velger til feldig}}$$

$$R = 5 + S, \quad \text{der } S \sim \text{Binom}(n=10, q=\tfrac{1}{2})$$

X = antall poeng

$$= 4R - 15 = 4(5+S) - 15$$

$$= 20 + 4S - 15 = \underline{4S + 5}$$

(se oppgaveark 3)

a) $p(R=15) = p(S=10) = \binom{10}{10} \cdot (\tfrac{1}{2})^{10} \cdot (\tfrac{1}{2})^{0} = \underline{(\tfrac{1}{2})^{10}}$

$$\simeq \underline{0.0010}$$

$p(R=14) = p(S=9) = \binom{10}{9} (\tfrac{1}{2})^{9} (\tfrac{1}{2}) = \underline{10 \cdot (\tfrac{1}{2})^{10}}$

$$\simeq \underline{0.0098}$$

$p(R=13) = p(S=8) = \binom{10}{8} \cdot (\tfrac{1}{2})^{8} \cdot (\tfrac{1}{2})^{2} = \underline{45 \cdot (\tfrac{1}{2})^{10}}$

$$\simeq \underline{\underline{0.0439}}$$

b) $E(R) = E(5+S) = 5 + E(S) = 5 + 10 \cdot \frac{1}{2} = \underline{\underline{10}}$

$\mathrm{Var}(R) = \mathrm{Var}(S+5) = \mathrm{Var}(S) = 10 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{2}$

$\Rightarrow \underline{\underline{\sigma_R}} = \sqrt{5/2} \simeq \underline{\underline{1.58}}$

c) $E(X) = E(4R-15) = 4E(R) - 15 = 4 \cdot 10 - 15$

$= \underline{\underline{25}}$

$\mathrm{Var}(X) = \mathrm{Var}(4R-15) = 4^2 \cdot \mathrm{Var}(R)$

$= 16 \cdot \frac{5}{2} = 40 \Rightarrow \sigma_X = \underline{\underline{\sqrt{40}}}$

$\simeq \underline{\underline{6.32}}$

d) $p(X \geq 37) = p(R \geq 13) = p(S \geq 8)$

$= p(S=8) + p(S=9) + p(S=10)$

$p(S=8) = \binom{10}{8} (1/2)^8 \cdot (1/2)^2 = 45 \cdot (1/2)^{10}$

$p(S=9) = \binom{10}{9} \cdot (1/2)^9 \cdot (1/2) = 10 \cdot (1/2)^{10}$

$p(S=10) = \binom{10}{10} \cdot (1/2)^{10} \cdot (1/2)^0 = 1 \cdot (1/2)^{10}$

$\Rightarrow p(X \geq 37) = (45+10+1) \cdot (1/2)^{10} = 56 \cdot (1/2)^{10}$

$\simeq \underline{\underline{0.055}}$

$\boxed{\text{Merk: } p(S=8) = p(S=2) \quad p(S=10) = p(S=0) \\ p(S=9) = p(S=1) \quad \text{pga symmetri}}$

Et kast: $S = \{5,6\}$ $p(S) = 2/6 = 1/3$

$F = \{1,2,3,4\}$ $p(F) = 4/6 = 2/3$

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a) $p(X=2) = p(FS) = 2/3 \cdot 1/3 = \underline{\underline{2/9}}$

b) $p(X>2) = 1 - p(X \le 2) = 1 - p(X=1) - p(X=2)$

$= 1 - p(S) - p(FS)$

$= 1 - 1/3 - 2/9 = \dfrac{9-3-2}{9} = \underline{\underline{4/9}}$

c) d): Kan bruke geometrisk fordeling

(ikke gjennomgått i år)

4. X = antall jenter som trekkes ut

a) $p(X=2) = \dfrac{\binom{13}{2}\cdot\binom{12}{3}}{\binom{25}{5}}$

antall måter å trekke ut 2 jenter og 3 gutter

antall måter å trekke ut 5 personer

$= \dfrac{78 \cdot 220}{53.130} \approx \underline{\underline{0.323}}$

b) $p(X>2) = 1 - p(X \le 2)$ kritisk fordeling

$= 1 - p(X=0) - p(X=1) - p(X=2)$

$$p(X=0) = \frac{\binom{13}{0}\binom{12}{5}}{\binom{25}{5}} = \frac{792}{53130} \simeq \underline{0.0149}$$

$$p(X=1) = \frac{\binom{13}{1}\cdot\binom{12}{4}}{\binom{25}{5}} = \frac{13\cdot 495}{53.130} \simeq \underline{0.121}$$

$$p(X=2) = \frac{\binom{13}{2}\binom{12}{3}}{\binom{25}{5}} = \frac{78\cdot 220}{53.130} \simeq \underline{0.323}$$

$$p(X>2) \simeq 1 - 0.0149 - 0.121 - 0.323$$
$$= \underline{\underline{0.541}}$$

c) d) Kan bruke <u>hypergeometrisk fordeling</u> (ikke gjennomsnitt i år)

5. X = ant oppringninger per 10 min

$X \sim \text{Poisson}(\lambda = 5)$

$\frac{30}{6} = 5$

siden

$\frac{60\text{ min}}{6} = 10\text{ min}$

a)

$$p(X=2) = \frac{5^2}{2!} e^{-5} = \frac{25}{2} e^{-5}$$

$$\simeq \underline{\underline{0.084}}$$

b) $p(x > 2) = 1 - p(x \leq 2) = 1 - p(0) - p(1) - p(2)$

$$= 1 - \frac{5^0}{0!} e^{-5} - \frac{5^1}{1!} e^{-5} - \frac{5^2}{2!} e^{-5}$$

$$= 1 - e^{-5}\left(1 + 5 + \frac{25}{2}\right) = 1 - 18.5\, e^{-5} \simeq \underline{\underline{0.875}}$$

c) $E(x) = \lambda = \underline{\underline{5}}$

d) $Var(x) = \lambda = 5 \Rightarrow \sigma_x = \underline{\underline{\sqrt{5}}} \simeq \underline{\underline{2.24}}$

formler for forventning / varians av en Poisson-fordelt variabel

$\boxed{E(X) = \lambda \qquad Var(x) = \lambda}$

6. X : $\mu_X = 3$ $\sigma_X = 1.25$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-3}{1.25}$$

Chebychevs ulikhet: $\mu = 3$, $\sigma = 1.25$

$$p(|X-3| \geq k\sigma) \leq 1/k^2 \iff p(|X-3| < k\sigma) \geq 1 - \frac{1}{k^2}$$

eller

$$p(|Z| \geq k) \leq 1/k^2 \iff p(|Z| < k) \geq 1 - \frac{1}{k^2}$$

$1 \leq X \leq 5$ kan skrives $1 \leq X \leq 5$

$$-2 = 1-3 \leq X-3 \leq 5-3 = 2$$

$$\frac{-2}{1.25} \leq \frac{X-3}{1.25} \leq \frac{2}{1.25}$$

$$-1.6 \leq Z \leq 1.6$$

$$p(1 \leq x \leq 5) = p(-1.6 \leq Z \leq 1.6)$$

$$= p(|Z| \leq 1.6) \geq 1 - \frac{1}{1.6^2} \simeq \underline{\underline{0.61}}$$

$k = 1.6$

Chebychevs ulikhet