Løsning: Oppgavesett 11

Monday, 23 March 2020 10:08

1.

a) Konfidensintervall for μ,
σ ukjent: T-intervall

$$\bar{x} \pm t_{\alpha/2}^{n-1} \cdot s/\sqrt{n}$$

$$\bar{x} = \frac{1}{9}(103 + \dots + 107) \simeq 99.56$$

kalk.

$$s = \sqrt{\frac{1}{8}\left((103-99.56)^2 + \dots + (107-99.56)^2\right)}$$

$$\simeq 7.50$$

kalk.

$$n = 9 \quad \alpha = 0.10 \quad t_{0.05}^{8} \simeq 1.860$$

$$\Downarrow$$

$$99.56 \pm 1.86 \cdot 7.50/\sqrt{9} \simeq 99.56 \pm 4.65$$

$$[94.9,\ 104.2]$$

b) Konfidensintervall for σ^2

$$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}},\ \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right]$$

$$n-1 = 8, \quad s^2 \simeq 7.50^2 \simeq 56.3$$

n-1=8 frihetsgr.

$$\chi^2_{0.05} = 15.51 \qquad \chi^2_{0.95} = 2.73$$

Tabell E.6

$$\left[\frac{8 \cdot 56.3}{15.51},\ \frac{8 \cdot 56.3}{2.73}\right] = [29.0,\ 164.9]$$

c) Forutsetninger:

$X_1, \dots, X_n$: uavhengige, normalfordelte målinger med $X_i \sim N(\mu, \sigma^2)$

2.

a) Konfidensintervall for p:

$n = 200$
$X = 4$

$\hat{p} = X/n = 4/200 = 0.02$

$n \cdot \hat{p} \cdot (1-\hat{p}) = 200 \cdot 0.02 \cdot 0.98$
$\simeq 3.92$

Vi burde hatt $n\hat{p}(1-\hat{p}) \geq 5$ for at n skulle vært stor nok til X er tilnærmet normalfordelt

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$$

$$\simeq \underbrace{0.02}_{\hat{p}} \pm \underbrace{1.96}_{z_{\alpha/2} \text{ (kalk.)}} \sqrt{\frac{0.02 \cdot 0.98}{200}}$$

$$\simeq 0.02 \pm 0.019$$

$$[0.001, 0.039] = [0.1\%, 3.9\%]$$

b) Forutsetninger:

X binomisk fordelt $\text{Binom}(n, p)$, og

n stor $(n\hat{p}(1-\hat{p}) \geq 5)$ slik at X

er tilnærmet normalfordelt

Det siste er ikke oppfyllt her.

c) Nytt utvalg: $\hat{p} = 9/400 = 0.0225$

$$n \cdot \hat{p}(1-\hat{p}) \simeq 8.8 \geq 5 \text{ ok}$$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.0225 \pm 1.96 \sqrt{\frac{0.0225 \cdot 0.9775}{400}}$$

$$= 0.0225 \pm 0.0145$$

$$[0.008, 0.037] = [0.8\%, 3.7\%]$$

d)

Ja, siden $\hat{p}(1-\hat{p}) \cdot n \geq 5$ (se ovenfor).

95% sikkert at virkelig andel er i intervallet 0,8% − 3,7%, og spesielt mindre enn 5%.

3. Se Eksamen 06/2019, Oppg. 2.