Kombinatorikk

Binomialkoeffisienter

$n! = n(n-1)\cdots 2\cdot 1 \qquad 0! = 1$

$\binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad \binom{n}{0} = 1$

Urnemodell

Ordnet, med tilbakelegging	n^r
Ordnet, uten tilbakelegging	$\frac{n!}{(n-r)!}$
Uordnet, uten tilbakelegging	$\binom{n}{r}$
Uordnet, med tilbakelegging	$\binom{n+r-1}{r}$

Sannsynlighetsregning

Mengder

Distributiv lov	$A\cap(B\cup C) = (A\cap C)\cup(B\cap C)$
de Morgans lover	$(A\cup B)^c = A^c\cap B^c$
	$(A\cap B)^c = A^c\cup B^c$

Regneregler for sannsynlighet

$p(A^c) = 1 - P(A)$
$p(A\cup B) = p(A) + p(B) - p(A\cap B)$
$p(A|B) = p(A\cap B)/p(B)$

$$p(B|A) = \frac{p(A|B)\cdot p(B)}{p(A)}$$

Stokastiske variabler

Diskrete fordelinger

$E(X) = \sum x\cdot p(x) = x_1\,p(x_1) + x_2\,p(x_2) + \ldots$
$E[g(X)] = \sum g(x)\cdot p(x) = g(x_1)\,p(x_1) + g(x_2)\,p(x_2) + \ldots$
$\mathrm{Var}(X) = E[(X-\mu)^2] = E(X^2) - E(X)^2$

Binomisk fordeling $\quad X \sim \mathrm{Binom}(n,p)$

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,\ldots,n$$
$E(X) = np$
$\mathrm{Var}(X) = np(1-p)$

Poisson-fordeling $\quad X \sim \mathrm{Poisson}(\lambda)$

$$p(x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0,1,2,\ldots$$
$E(X) = \lambda$
$\mathrm{Var}(X) = \lambda$

Kontinuerlige fordelinger

$$p(a\le X\le b) = \int_a^b f(x)\,\mathrm{d}x$$
$$E(X) = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x$$
$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\,\mathrm{d}x$$
$\mathrm{Var}(X) = E[(X-\mu)^2] = E(X^2) - E(X)^2$

Normalfordeling $\quad X \sim \mathrm{N}(\mu,\sigma^2)$

$E(X) = \mu$
$\mathrm{Var}(X) = \sigma^2$
$X \sim (\mu,\sigma^2) \quad \Rightarrow \quad Z = \frac{X-\mu}{\sigma} \sim \mathrm{N}(0,1)$

Eksponential-fordeling $\quad X \sim \mathrm{Exp}(\lambda)$

$E(X) = 1/\lambda$
$\mathrm{Var}(X) = 1/\lambda^2$

Simultane fordelinger

$f_X(x) = \sum_y f(x,y), \quad f_Y(y) = \sum_x p(x,y)$
$E[g(X,Y)] = \sum_{x,y} g(x,y)p(x,y)$
$\mathrm{Cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - E(X)E(Y)$

$$\rho = \rho_{X,Y} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X\,\sigma_Y}$$

Regneregler for stokastiske variabler

$E(aX+bY+c) = aE(X) + bE(Y) + c$
$\mathrm{Var}(aX+bY+c) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) + 2ab\,\mathrm{Cov}(X,Y)$

$\mathrm{Cov}(Y,X) = \mathrm{Cov}(X,Y)$
$\mathrm{Cov}(X,X) = \mathrm{Var}(X)$
$\mathrm{Cov}(aX+bY+c,Z) = a\,\mathrm{Cov}(X,Z) + b\,\mathrm{Cov}(Y,Z)$

$-1 \le \rho_{X,Y} \le 1$
$\rho > 0 \quad \Leftrightarrow \quad Y = aX+b$ med $a > 0$
$\rho < 0 \quad \Leftrightarrow \quad Y = aX+b$ med $a < 0$

Normaltilnærming av binomisk fordeling

$X \sim \mathrm{Binom(n,p)}$ med n stor og p nær 1/2 har tilnærming
$Y \sim N(\mu,\sigma^2)$ med $\mu = np$ og $\sigma^2 = np(1-p)$:

$$p(a\le X\le b) \cong p(a-0.5 \le Y \le b+0.5)$$