	Emne	Lærebok	Oppgaver
1	Repetisjon og oppgavegjennomgang		
2	Arealberegning og bestemte integral	[E] 5.6	5.6.3 - 5.6.5

1 Repetisjon

$\int \frac{f(x)}{g(x)}\,dx$: Metoder
- polynomdivisjon
- substitusjon
- delbrøkoppspaltning

Eks: $\frac{x+1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$

Oppgaver 27-28

1d) $\int \frac{x^2}{4-x^2}\,dx = \int -1 + \frac{4}{4-x^2}\,dx$ delbrøk

$$\frac{x^2}{4-x^2} = \frac{A}{2-x} + \frac{B}{2+x} \quad \Big| \cdot (2-x)(2+x)$$

$$x^2 = A(2+x) + B(2-x)$$

~~$x=-2:\quad 4 = B\cdot 4 \quad B=1$~~

~~$x=2:\quad 4 = A\cdot 4 \quad A=1$~~

$$x^2 = 2A + Ax + 2B - Bx$$

$$x^2 = (A-B)x + (2A+2B)$$

går ikke

5b) $\int x \ln(1-x)\,dx = \int x \cdot \ln u \, dx = \int (1-u) \ln u \cdot \frac{1}{(-1)}\,du$

$$\boxed{\begin{array}{l} u = 1-x \\ du = -1\cdot dx \end{array}} \quad \begin{array}{l} \Rightarrow x = 1-u \\ \Rightarrow dx = \frac{1}{-1}\,du \end{array}$$

$= \int \underbrace{(u-1)}_{=v'} \underbrace{\ln u}_{=w}\,du = \underbrace{(\tfrac{1}{2}u^2 - u)}_{=v} \underbrace{\ln u}_{=w} - \int \underbrace{(\tfrac{1}{2}u^2 - u)}_{=v} \cdot \underbrace{\tfrac{1}{u}}_{=w'}\,du$

$\frac{1}{2}(u-1)^2 \rightarrow$

$$\boxed{\begin{array}{ll} v = \frac{1}{2}u^2 - u & w = \ln u \\ v' = u-1 & w' = 1/u \end{array}}$$

$= (\tfrac{1}{2}u^2 - u)\ln u - \int \tfrac{1}{2}u - 1\,du = \underline{(\tfrac{1}{2}u^2 - u)\ln u - \tfrac{1}{4}u^2 + u + C}$

$= \underline{\underline{\left[\tfrac{1}{2}(1-x)^2 - (1-x)\right]\ln(1-x) - \tfrac{1}{4}(1-x)^2 + 1 - x + C}}$

7a) $\int 2x^3 e^{-x^2}\,dx = \int \cancel{2}x\cancel{x}^2 e^u \frac{1}{-\cancel{2}\cancel{x}}\,du = \int -x^2 e^u\,du$

$$\boxed{\begin{array}{l} u = -x^2 \\ du = -2x\,dx \end{array}} \quad dx = \frac{1}{-2x}\,du \qquad = \int u e^u\,du$$

$= \ldots$ delvis $= u e^u - e^u + C = \underline{-x^2 e^{-x^2} - e^{-x^2} + C}$

b) $\int \sqrt{x}\, e^{\sqrt{x}}\,dx = \int \sqrt{x}\, e^u \cdot 2\sqrt{x}\,du = \int 2x e^u\,du$

$$x = u^2 \Leftarrow \boxed{\begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx \end{array}} \quad \Rightarrow dx = 2\sqrt{x}\cdot du \qquad = \int 2u^2 e^u\,du$$

$= \ldots$ delvis (2 steg) $= 2u^2 e^u - 4u e^u + 4e^u + C$

$= \underline{2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4 e^{\sqrt{x}} + C}$

7c) $$\int \frac{\sqrt{x}+1}{1-\sqrt{x}}\,dx = \int \frac{\sqrt{x}+1}{u}(-2\sqrt{x})\,du$$

$$\sqrt{x} = 1-u \leftarrow \boxed{\begin{array}{c} u = 1-\sqrt{x} \\ du = -\frac{1}{2\sqrt{x}}dx \end{array}} \quad dx = -2\sqrt{x}\,du$$

$$= \int \frac{-2\sqrt{x}(\sqrt{x}+1)}{u}\,du = \int \frac{-2(1-u)(1-u+1)}{u}\,du$$

$$= \int \frac{-2(1-u)(2-u)}{u}\,du = -2\int \frac{2-3u+u^2}{u}\,du$$

$$= -2\left(2\ln|u| - 3u + \tfrac{1}{2}u^2\right) + C = \underline{-4\ln|1-\sqrt{x}| + 6(1-\sqrt{x}) - (1-\sqrt{x})^2 + C}$$

8b) $$\int_{x=0}^{x=1} 15x\sqrt{x+1}\,dx = \int 15x\sqrt{u}\,du = \int_{u=1}^{u=2} 15(u-1)\sqrt{u}\,du$$

$$\boxed{\begin{array}{c} u = x+1 \\ du = dx \end{array}} \quad x = u-1$$

$$= \int_1^2 15u^{3/2} - 15u^{1/2}\,du = \Big[15\cdot\underbrace{\tfrac{2}{5}}_{\frac{1}{5/2}}\cdot u^{5/2} - 15\cdot\tfrac{2}{3}u^{3/2}\Big]_1^2$$

$$= \Big[6u^{5/2} - 10u^{3/2}\Big]_1^2 = \left(6\cdot 2^{5/2} - 10\cdot 2^{3/2}\right) - (-4)$$

$$= \underline{6\cdot 2^{5/2} - 10\cdot 2^{3/2} + 4} = 6\cdot 2^2\cdot\sqrt{2} - 10\cdot 2\cdot\sqrt{2} + 4$$

$$= 24\sqrt{2} - 20\sqrt{2} + 4 = \underline{4\sqrt{2}+4}$$

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9. $\displaystyle\int_{-1}^{1} \frac{e^x}{e^x+1}\,dx = \int \frac{\cancel{e^x}}{u} \cdot \frac{1}{\cancel{e^x}}\,du$

$x=1 : u = e+1$
$x=-1 : u = e^{-1}+1$

$$\boxed{\begin{aligned} u &= e^x+1 \\ du &= e^x\,dx \end{aligned}}$$

$\cancel{dx} \; dx = \frac{1}{e^x}\,du$

$$= \int_{e^{-1}+1}^{e+1} \frac{1}{u}\,du = \Big[\ln|u|\Big]_{e^{-1}+1}^{e+1}$$

$$= \underline{\ln(e+1) - \ln(\overset{1/e}{\overset{\shortparallel}{e^{-1}}}+1)}$$

$\ln(ab) = \ln(a)+\ln(b)$
$\ln(a/b) = \ln(a)-\ln(b)$
$\ln(a^n) = n\cdot\ln(a)$

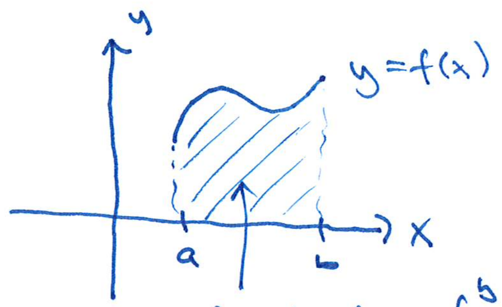
$$= \ln\frac{e+1}{e^{-1}+1} = \ln\frac{e+1}{\frac{1}{e}+1}\frac{\cdot e}{\cdot e} = \ln\frac{e\cancel{(e+1)}}{\cancel{1+e}} = \ln e = \underline{\underline{1}}$$

② Arealberegning

Teorem

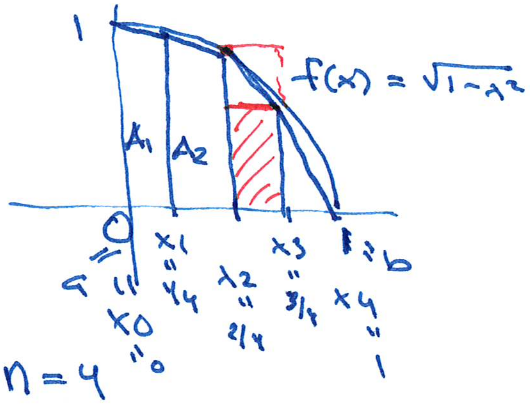
Hvis $f(x)$ er kontinuerlig på et intervall $[a,b]$ og $f(x) \geq 0$ for alle x i $[a,b]$, så er arealet under grafen til f på intervallet $[a,b]$

$$\boxed{A = \int_a^b f(x)\,dx}$$



Areal $A = \int_a^b f(x)\,dx$.

Eks:



$n = 4$

$\Delta x = \frac{b-a}{n} = 1/4$

$x_i = \Delta x \cdot i = \frac{1}{4} \cdot i$

$A_3 \simeq \Delta x \cdot f(x_3)$
$= \frac{1}{4} \cdot \sqrt{1-(3/4)^2}$

eller

$A_3 \simeq \Delta x \cdot \frac{f(x_2)+f(x_3)}{2}$

Merk: $y = \sqrt{1-x^2}$
$y^2 = 1-x^2$
$x^2+y^2 = 1 \leftarrow$ sirkel m/radius $= 1$

Areal $= \pi \cdot 1^2/4 = \pi/4$

Areal under grafen til f på [0,1]

$A = A_1 + A_2 + A_3 + A_4$

$\simeq \frac{1}{4} \cdot \frac{f(x_0)+f(x_1)}{2} + \frac{1}{4} \cdot \frac{f(\overset{1/4}{\overset{\shortparallel}{x_1}})+f(x_2)}{2}$

$+ \frac{1}{4} \cdot \frac{f(x_2+x_3)}{2} + \frac{1}{4} \frac{f(x_3)+f(x_4)}{2}$

$= \frac{1}{4} \cdot \frac{1}{2} \Big(\sqrt{1} + 2\sqrt{1-(1/4)^2}$
$+ 2\sqrt{1-(2/4)^2}$
$+ 2\sqrt{1-(3/4)^2} + \sqrt{0} \Big)$

$\simeq \underline{0.749}$

Riemannsum

$\sum_{i=1}^{n} \Delta x \cdot \frac{f(x_{i-1})+f(x_i)}{2}$ eller $\boxed{\sum_{i=1}^{n} f(x_i) \cdot \Delta x} \rightsquigarrow \int_a^b f(x)\,dx$

$\downarrow n \to \infty$ $\quad$ $n \to \infty$

Arealet under grafen til f

$\int = \text{sum!}$