Plan

1 Constrained optimization
2 Extreme value theorem

No working mic's in this room, sorry.

Course paper: Solutions are posted.

① Constrained optimization

max/min $\underbrace{f(x,y)=x^2+y^2}_{\text{objective fn.}}$ when $\underbrace{0\leq x,y\leq 1}_{\text{constraints}}$ constraind optimization

$D=\{(x,y):\ 0\leq x,y\leq 1\}$ in $\mathbb{R}^2$
set of admissible pts.

max/min $f(x,y)=x^2+y^2$ unconstraind optimization

Unconstrained max/min $f(x,y)$

Candidate pts:

i) Stat. pts.: $f'_x=f'_y=0$

ii) Other critical pts: f'_x or f'_y not defn.

iii) boundary pts of D_f

(local) classification:

Second derivative test
→ local max, local min, saddle pt.

are any of these pts (global) max/min

Constrained max/min $f(x,y)$ when (x,y) is in D

Candidate pts:

i) interior stat. pts: $f'_x=f'_y=0$

ii) interior critical pts: f'_x or f'_y does not exist

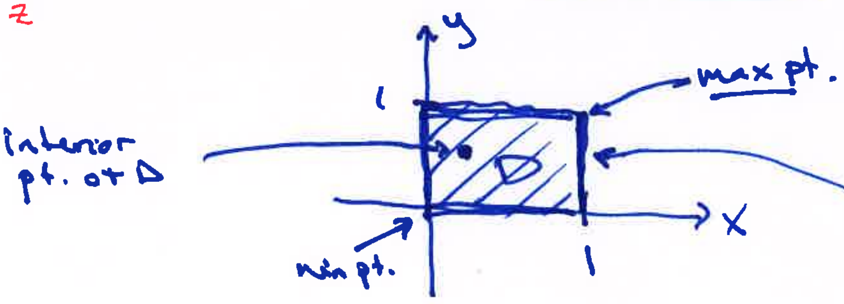
iii) Boundary pts of D, where $D=\{(x,y):$ satisfy all constraints$\}$

EVT: if D is compact (closed + bounded) then there is a max/min.

Determine whether candidate pts are (global) max/min: Use EVT if D is compact.

Ex: max/min $f(x,y)=x^2+y^2$ when $\boxed{0 \le x,y \le 1}$

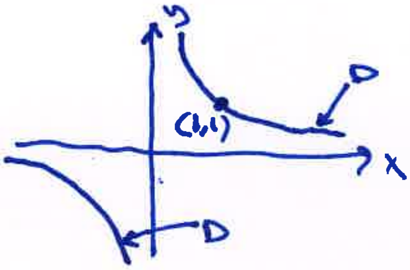
z



∂D = Boundary of D
= four sides of the square

Ex: min $f(x,y)=x^2+y^2$ when $xy=1$

no interior pts.

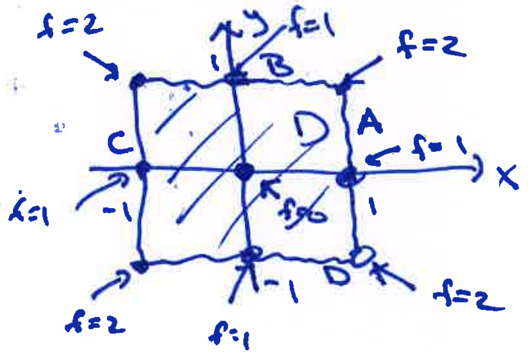


$xy=1 \Rightarrow y=1/x$

∂D = Boundary pts of D = D
(all pts. on D)

Ex: max/min $f(x,y) = x^2+y^2$ when $-1 \le x,y \le 1$

Candidate pts:



i) $f'_x = 2x = 0 \Rightarrow x = 0$

$f'_y = 2y = 0 \qquad y = 0$

$(0,0)$ interior pt of $D \Rightarrow$

Candidate i): $(x,y) = \underline{(0,0)}$

$f(0,0) = 0$

ii) other critical pts in the interior of D: $\underline{\text{none}}$

iii) Boundary pts: ∂D = four sides

- A: $x=1$, $-1 \le y \le 1$
- B: $y=1$, $-1 \le x \le 1$
- C: $x=-1$; $-1 \le y \le 1$
- D: $y=-1$; $-1 \le x \le 1$

EVT: f cont: ✓ (ok)
D compact: (ok) closed ✓ bounded ✓ } there is a max/min

$\Rightarrow$ there is a max and a min among the candidate pts.

Candidate pts: compare values

i) $(0,0)$: $f(0,0) = \underline{0}$

ii) none

iii)

A: $f(1,y) = 1+y^2$, $-1 \le y \le 1$
max: $f(1,1) = f(1,-1) = 2$ min: $f(1,0) = \underline{1}$

B: $f(x,1) = x^2+1$, $-1 \le x \le +1$
max: $f(1,1) = \underline{2}$, $f(-1,1) = \underline{2}$ min: $f(0,1) = \underline{1}$

C: $f(-1,y) = 1+y^2$, $-1 \le y \le 1$ (similar with A)
max: $f(-1,1) = f(-1,-1) = \underline{2}$
min: $f(-1,0) = \underline{1}$

D: $f(x,-1) = x^2+1$, $-1 \le x \le 1$ (similar with B)
max: $f(1,-1) = f(-1,-1) = 2$
min: $f(0,-1) = 1$

Alt: Side A ($x=1$) $f(1,y) = 1+y^2$, $-1 \le y \le 1$

$(1+y^2)' = 2y$

$f(1,-1) = 2$ $f(1,1) = 2$ → max on A

$f(1,0) = 1$ → min on A

Conclusion: D is compact, so there is max/min.

Highest value among cand. pts: $f_{max} = \underline{\underline{2}}$

(at the max. pts. $(1,1), (-1,1), (1,-1), (-1,-1)$)

Lowest value — " —: $f_{min} = \underline{\underline{0}}$

(at the min. pts $(0,0)$)

② Extreme value theorem:

$f(x,y)$ is a continuous fn. on a set D in $\mathbb{R}^2$

Extreme Value Theorem: (EVT)

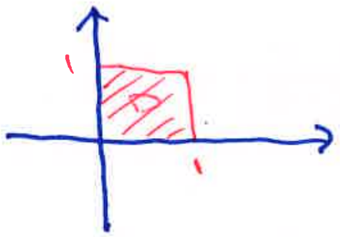
If f is a continuous fn. on a compact set D in $\mathbb{R}^2$, then f has a maximum and a minimum on D.

Defn.: A subset D of $\mathbb{R}^2$ is called compact if it is closed and bounded.

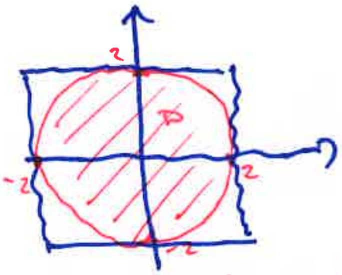
A subset D of $\mathbb{R}^2$ is closed if all boundary pts of D are included in D.

$= \quad \leq \quad \geq \quad =$ yes $\qquad < \quad > \quad =$ no

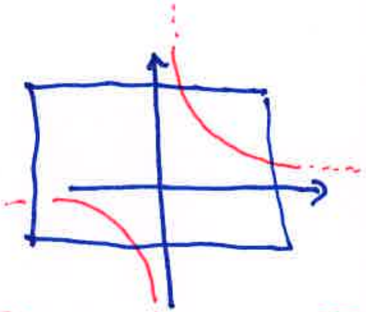
A subset D of $\mathbb{R}^2$ is bounded if there is a rectangle in $\mathbb{R}^2$ (with finite sides) that includes all of D



$D: 0 \leq x,y \leq 1$ yes



$D: x^2+y^2 \leq 4$ yes



$D: xy=1$ no