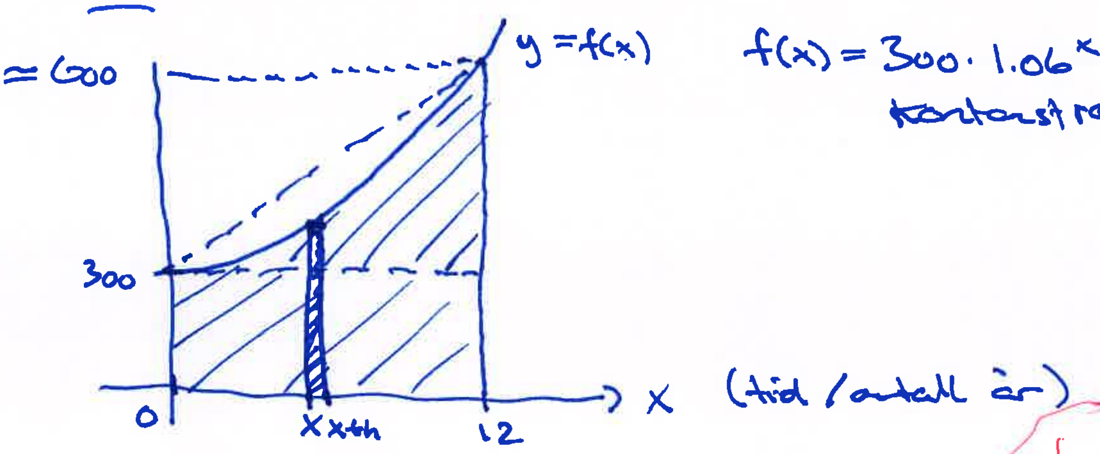
Plan

1. Økonomiske anvendelser av bestemte integral
2. Partiellderivasjon

① Økonomiske anvendelser av integrasjon

(a) Kontinuerlige kontantstrømmer

Eks:



$f(x) = 300 \cdot 1.06^x$ kontantstrøm (MNOK/år)

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{1}{\ln(a)} a^x + C$

Samlet kontantstrøm:

$$\int_0^{12} f(x)\,dx = \int_0^{12} 300 \cdot 1.06^x\,dx = 300\left[\frac{1}{\ln(1.06)} \cdot 1.06^x\right]_0^{12}$$

$$= \frac{300}{\ln(1.06)} \cdot \left(1.06^{12} - 1\right) \simeq \underline{5.211} \text{ MNOK}$$

Samlet kontantstrøm: $\displaystyle\int_{t_0}^{t_1} f(x)\,dx$

tidsperiode: $[t_0, t_1]$
kont. kontantstrøm: $f(x)$

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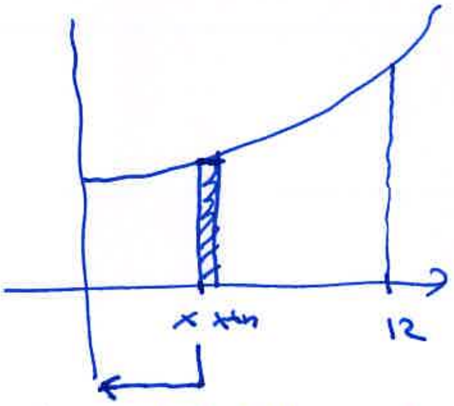
Nåverdi av kontantstrøm:



$f(x) = 300\cdot 1.06^x$

$$1.06^x = \left(e^{\ln 1.06}\right)^x = e^{\ln(1.06)x}$$

diskonteringsrate: $r = 10\%$

kontinuerlig diskontering $\quad \cdot \ \dfrac{1}{e^{rx}} = e^{-rx} = e^{-0.10\cdot x}$

Nåverdi: $\displaystyle \int_0^{12} f(x)\cdot e^{-rx}\,dx = \int_0^{12} 300\cdot 1.06^x \cdot e^{-0.10x}\,dx$

$$= \int_0^{12} 300\, \underline{e^{\ln(1.06)x}\cdot e^{-0.10x}}\,dx$$

$$= 300\int_0^{12} e^{\ln(1.06)x - 0.10x}\,dx$$

$$\boxed{\begin{aligned} u &= \ln(1.06)x - 0.10x \\ du &= \underbrace{\left(\ln(1.06) - 0.10\right)}_{u'}dx \end{aligned}}$$

$$= 300\int_*^* e^u \frac{1}{\ln 1.06 - 0.10}\,du$$

$$= \frac{300}{\ln 1.06 - 0.10}\cdot\left[e^u\right]_*^* = \frac{300}{\ln 1.06 - 0.10}\left[e^{\ln(1.06)x - 0.10x}\right]_0^{12}$$

$$= \frac{300}{\ln(1.06) - 0.10}\left(e^{\ln(1.06)\cdot 12 - 1.2} - 1\right)$$

$$= \underbrace{\frac{300}{\ln(1.06) - 0.10}}_{-7.187}\underbrace{\left(\frac{1.06^{12}}{e^{1.2}} - 1\right)}_{-0.394} \simeq \underline{\underline{2.832}} \text{ MNOK}$$

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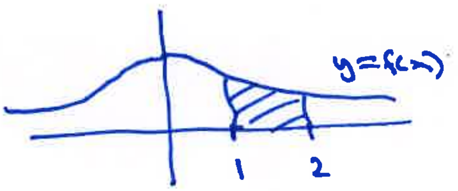
Formel for nåverdi av kont. kontantstrøm: $\int_{t_0}^{t_1} f(x) \cdot e^{-rx}\, dx$

tidsintervall: $[t_0, t_1]$
kont. kontantstrøm: $f(x)$
kont. diskonteringsrente: r

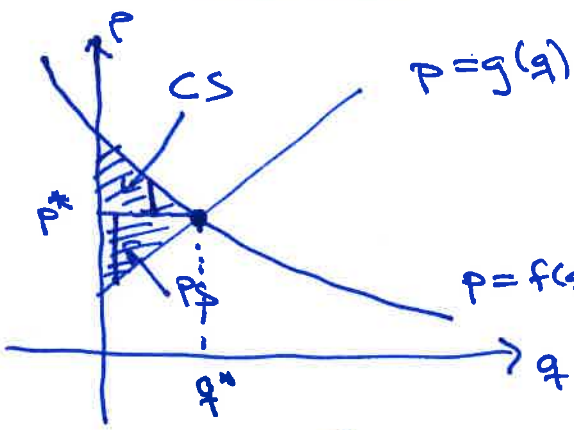
(b) Sannsynligheter (kontinuerlig stokastisk variabel)

Eks: $f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$ tetthetsfunksjon til standard normalfordeling



$$p(1 \leq X \leq 2) = \int_1^2 f(x)\, dx$$
$$= \int_1^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

(c) Konsument / produsent overskudd



$p = g(q)$ (omvendt) tilbudsfunksjon

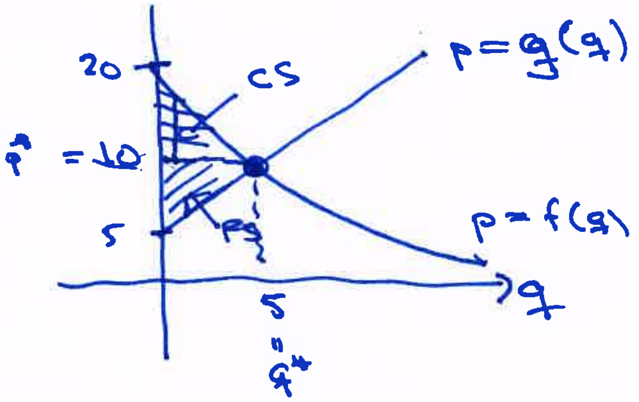
$p = f(q)$ (omvendt) etterspørselsfn.

$$CS = \int_0^{q^*} f(q) - p^*\, dq$$
$$PS = \int_0^{q^*} p^* - g(q)\, dq$$

Eks: $f(q) = \dfrac{100}{q+5}$

$g(q) = q+5$

$$f(q) = g(q)$$
$$\frac{100}{q+5} = q+5$$
$$100 = (q+5)^2$$
$$\Rightarrow q+5 = \pm\sqrt{100} = 10$$
$$\underline{q = 5}$$



$$PS = \frac{5\cdot 5}{2} = \frac{25}{2} = \underline{12.5}$$

$$\left(= \int_0^5 10 - (q+5)\, dq = \int_0^5 5 - q\, dq = \left[5q - \tfrac{1}{2}q^2\right]_0^5 = \left(5\cdot 5 - \tfrac{1}{2}\cdot 5^2\right) - 0 = \underline{\underline{12.5}} \right)$$

$$CS = \int_0^5 \frac{100}{q+5} - 10\, dq$$

$$= \left[100\cdot \ln(q+5) - 10q\right]_0^5$$

$$= \left(\underline{100 \ln(10)} - 50\right) - \left(\underline{100 \ln(5)} \cancel{-0}\right)$$

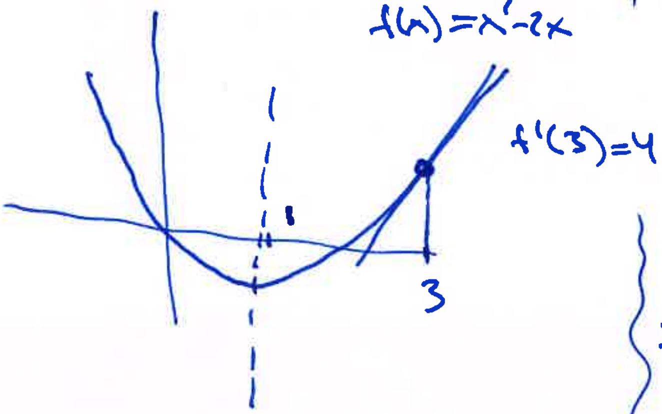
$$= 100\cdot(\ln 10 - \ln 5) - 50 = 100\cdot \ln\left(\frac{10}{5}\right) - 50$$

$$= \underline{\underline{100 \ln(2) - 50}} \simeq 19$$

② Partiellderivasjon

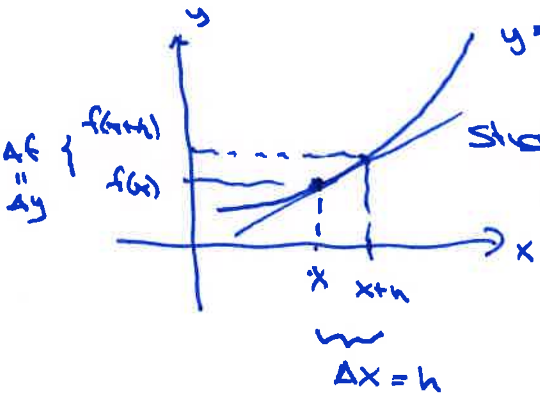
Eks: $f(x) = x^2 - 2x$

$$f'(x) = (x^2-2x)' = \underline{\underline{2x-2}}$$



Leibniz' notasjon:

$$\frac{dy}{dx} = \frac{df}{dx} = f'(x) = \lim_{h\to 0} \frac{\overbrace{f(x+h)-f(x)}^{\Delta y = \Delta f}}{\underbrace{h}_{\Delta x}}$$



Eks: $f(x,y) = \underline{x^2 + 4y^2 - 2x}$

$$f'_x(x,y) = 2x + 0 - 2$$

$$f'_y(x,y) = 0 + 8y + 0$$

$f'_x = \underline{2x-2}$ $\quad f'_x(2,1) = \underline{\underline{2}}$

$f'_y = \underline{8y}$ $\quad f'_y(2,1) = \underline{\underline{8}}$

Eks: $f(2,1) = 2^2 + 4\cdot 1^2 - 2\cdot 2 = \underline{\underline{4}}$

$z=f(x,y)$ grafen, 3d koord. system

stigningstallet til tangenten til $\underline{z=f(x,1)}$ i $\underline{x=2}$ $= \underline{\underline{2}}$

$z=f(x,1) = x^2-2x+4$

Leibniz' notasjon:

$$\frac{df}{dx} = f'_x$$

$$\frac{df}{dy} = f'_y$$

Eks: $f(x,y) = x^2 - \underline{2xy} + 3y^2 - 2y + x - 4$

$f'_x(x,y) = f'_x = 2x - 2y \cdot 1 + 0 + 0 + 1 + 0 = \underline{2x - 2y + 1}$

$f'_y(x,y) = f'_y = 0 - 2x \cdot 1 + 6y - 2 + 0 = \underline{-2x + 6y - 2}$