Plan

① Determinanter og lineære system

Eks:
$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} = \underline{\underline{4}}$$

Alternativ metode for determinanter uha Gauss

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} \cdot (-1) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} \cdot (-1) = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{vmatrix} = \underline{+4}$$

a) En kvadratisk matrise på trappeform er øvre triangulær, og determinanten er produktet av tallene på diagonalen: $1 \cdot 1 \cdot (-2) \cdot (-2) = \underline{4}$

b) Hvis $A \to B$ er en elementær radoperasjon:

i) Hvis $A \to B$ er å bytte om to rader : $|B| = -|A|$

ii) Hvis $A \to B$ er å multiplisere en rad med $c \neq 0$: $|B| = c \cdot |A|$

$\boxed{\text{iii) Hvis } A \to B \text{ er å legge til et multippel av en rad til en annen rad : } |B| = |A|}$

Lineære system:

Spesialtilfelle: Anta at det er like mange ukjente som likninger lineære (altså $n \times n$ system) $\Rightarrow$ Koeff.-matrisen til det lineære systemet er $n \times n$ (kvadratisk)

Eks:
$$\boxed{\begin{aligned} x + y + z &= 3 \\ x + 2y + 4z &= 7 \\ x + 3y + 9z &= 13 \end{aligned}} \longrightarrow A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$$

3x3 lineært system

Koeff. matrisen til systemet er kvadratisk

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 1 \cdot 6 - 1 \cdot 5 + 1 \cdot 1 = \underline{2} \neq 0 \Rightarrow \boxed{\text{en løsning}}$$

Resultat:
Vi ser på et $n \times n$ lineært system med koeffisient-matrise A. Da har vi:

i) $|A| \neq 0 \Rightarrow$ det lineære systemet har (eksakt) en løsning

ii) $|A| = 0 \Rightarrow$ — " — har ingen løsninger eller uendelig mange løsninger

Kommentar: $|A| = 0 \Leftrightarrow |E| = 0$ når E er en trappeform for A.

$A \to \to \ldots \to E$ trappeform

$$E = \begin{pmatrix} e_{11} & * & \cdots & * \\ 0 & e_{22} & \cdots & * \\ \vdots & & \ddots & \vdots \\ 0 & 0 & & e_{nn} \end{pmatrix} : \quad |E| = e_{11} \cdot e_{22} \cdot \ldots \cdot e_{nn} = 0$$
$\Updownarrow$
minst en $e_{ii} = 0$

Eks:

$$A \to \cdots \to E = \begin{pmatrix} 1 & 0 & 7 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{pmatrix} \qquad |E| = 10 \neq 0$$

når $|A| \neq 0 \Rightarrow$ pivot i alle variabelkolonner $\Rightarrow$ én løsning

$$A \to \cdots \to E = \begin{pmatrix} 1 & 0 & 7 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

når $|A| = 0 \Rightarrow$ minst én variabelkolonne uten pivot $\Rightarrow$ ingen eller uendelig mange løsninger (avh. av siste kolonne)

$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 2 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 0 & 4 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 7 & 2 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

ingen løsning — uendelig mange løsninger

② Lineære system med parametre

Eks.

$$\boxed{\begin{aligned} x + y &= 4 \\ x + ay &= 6 \end{aligned}}$$

x, y: variable (ukjente)
a: parameter
⇓
skal finne x og y som uttrykk i a.

i) Gauss eliminasjon:

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & a & 6 \end{array}\right) \cdot(-1) \to \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & a-1 & 2 \end{array}\right)$$

$a \neq 1$:

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & a-1 & 2 \end{array}\right)$$

én løsning

$a = 1$:

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & 2 \end{array}\right)$$

ingen løsning.

$(a-1)y = 2$

$$y = \frac{2}{a-1}$$

$x + y = 4$

$$x = 4 - \frac{2}{a-1} = \frac{4(a-1) - 2}{a-1} = \frac{4a-6}{a-1}$$

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Løsning: $(x,y) = \left(\frac{4a-6}{a-1}, \frac{2}{a-1}\right)$, $a \neq 1$

ingen løsning, $a = 1$

ii) Alt: Via determinanter

$$\left.\begin{array}{l} x+y=4 \\ x+ay=6 \end{array}\right\} \quad \underset{2\times 2}{A = \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix}}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = 1\cdot a - 1\cdot 1 = \underline{a-1}$$

a) $|A| = 0$: $a-1=0$, $\underline{a=1}$ ← ingen eller uendelig mange løsn. siden $\underline{|A|=0}$

b) $|A| \neq 0$: $a \neq 1$ ← én løsning

a) $\underline{a=1}$: < ingen løsn / uendel. mange løsn.

$\underline{a=1}$: $\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & 1 & 6 \end{array}\right) \xrightarrow{]-1} \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & 2 \end{array}\right)$

ingen løsn.

b) $\underline{a \neq 1}$: én løsning

Kramer's regel: $(A|\underline{b}) = \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & a & 6 \end{array}\right)$

$$x = \frac{|A_x(\underline{b})|}{|A|} = \frac{\begin{vmatrix} 4 & 1 \\ 6 & a \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix}} = \underline{\underline{\frac{4a-6}{a-1}}}$$

$$y = \frac{|A_y(\underline{b})|}{|A|} = \frac{\begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix}} = \underline{\underline{\frac{2}{a-1}}}$$

$\Downarrow$

$$(x,y) = \left(\frac{4a-6}{a-1}, \frac{2}{a-1}\right), \ a \neq 1$$

Eks: $a=2$: $(x,y) = \underline{(2,2)}$

Kramer's regel:

Vi ser på et $n \times n$ lineært system med koeffisient-matrise A og utvidet matrise $(A | \underline{b})$. Hvis $|A| \neq 0$, så har vi

$$x_1 = \frac{|A_1(\underline{b})|}{|A|}$$

$$x_2 = \frac{|A_2(\underline{b})|}{|A|}$$

$$\vdots$$

$$x_n = \frac{|A_n(\underline{b})|}{|A|}$$

der $A_i(\underline{b})$ er matrisen vi får om vi bytter ut kolonne i fra A med $\underline{b}$.

Metode: Anta at vi har et lineært system som er $n \times n$, med koeff. matrise A.

i) Finn $|A|$ og avgjør når $|A| = 0$.

ii) I tilfellene med $|A| = 0$: Sett inn parameterverdier og bruk Gauss.

iii) I tilfellene med $|A| \neq 0$: Bruk Kramer's regel.

Eks:
$$\left.\begin{aligned} x+y+z &= 3 \\ x+ay+4z &= 7 \\ x+3y+az &= 13 \end{aligned}\right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & a & 4 & 7 \\ 1 & 3 & a & 13 \end{array}\right)$$

i) $$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 4 \\ 1 & 3 & a \end{vmatrix} = 1\cdot(a^2-12) - 1\cdot(a-4) + 1\cdot(3-a)$$
$$= a^2 - 12 - a + 4 + 3 - a = a^2 - 2a - 5$$

$|A|=0$: $a^2-2a-5=0$
$$a = \frac{2 \pm \sqrt{4+20}}{2} = \frac{2\pm\sqrt{24}}{2} = \frac{2\pm\sqrt{4}\cdot\sqrt{6}}{2}$$
$$= 1 \pm \sqrt{6}$$

ii) $a = 1+\sqrt{6}$, $a = 1-\sqrt{6}$: ingen eller uendelig mange løsninger

$a = 1+\sqrt{6}$:
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1+\sqrt6 & 4 & 7 \\ 1 & 3 & 1+\sqrt6 & 13 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & \sqrt6 & 3 & 4 \\ 0 & 2 & \sqrt6 & 10 \end{array}\right) \cdot \sqrt6/3$$
$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 2 & \sqrt6 & 4\sqrt6/3 \\ 0 & 2 & \sqrt6 & 10 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 2 & \sqrt6 & 4\sqrt6/3 \\ 0 & 0 & 0 & 10-4\sqrt6/3 \end{array}\right)$$

ingen løsning

$a = 1-\sqrt{6}$: tilsvarende regning med $a = 1-\sqrt6$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & \sqrt6 & 4\sqrt6/3 \\ 0 & 0 & 0 & 10+4\sqrt6/3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1-\sqrt6 & 4 & 7 \\ 1 & 3 & 1-\sqrt6 & 13 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -\sqrt6 & 3 & 4 \\ 0 & 2 & -\sqrt6 & 10 \end{array}\right) \cdot \frac{\sqrt6}{3} \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & \sqrt6 & 4\sqrt6/3 \\ 0 & 2 & -\sqrt6 & 10 \end{array}\right)$$

ingen løsning

iii) $\underline{a \neq 1 \pm \sqrt{6}}$: $|A| \neq 0$ $\Rightarrow$ én løsning

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & a & 4 & 7 \\ 1 & 3 & a & 13 \end{array}\right): \quad x = \frac{|A_1(b)|}{|A|} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 7 & a & 4 \\ 13 & 3 & a \end{vmatrix}}{a^2-2a-5}$$

$$= \frac{3(a^2-12)-(7a-52)+(21-13a)}{a^2-2a-5} = \underline{\frac{3a^2-20a+37}{a^2-2a-5}}$$

$$y = \frac{|A_2(b)|}{|A|} = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 7 & 4 \\ 1 & 13 & a \end{vmatrix}}{a^2-2a-5}$$

$$= \frac{(7a-52)-3(a-4)+(6)}{a^2-2a-5} = \underline{\frac{4a-34}{a^2-2a-5}}$$

$$z = \frac{|A_3(b)|}{|A|} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & a & 7 \\ 1 & 3 & 13 \end{vmatrix}}{a^2-2a-5}$$

$$= \frac{(13a-21)-(6)+3(3-a)}{a^2-2a-5} = \underline{\frac{10a-18}{a^2-2a-5}}$$

③ Vektorlikninger:

$\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_r$: n-vektorer

Defn: En lineærkombinasjon av $\underline{v}_1, \ldots, \underline{v}_r$ er et uttrykk på formen

$$c_1 \cdot \underline{v}_1 + c_2 \cdot \underline{v}_2 + \ldots + c_r \cdot \underline{v}_r$$

der $c_1, c_2, \ldots, c_r$ er vilkårlige skalarer (tall)

Eks: $\underline{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ $\underline{v}_2 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ $\underline{v}_3 = \begin{pmatrix} 4 \\ -1 \\ 6 \end{pmatrix}$

Problem: Er $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ en lineærkomb. av $\underline{v}_1, \underline{v}_2, \underline{v}_3$?

Finn det x_1, x_2, x_3 slik at:

vektor-likning

$$\boxed{x_1 \cdot \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + x_3 \begin{pmatrix} 4 \\ -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}}$$

$$\begin{pmatrix} x_1 \\ 0 \\ 4x_1 \end{pmatrix} + \begin{pmatrix} 3x_2 \\ -x_2 \\ 2x_2 \end{pmatrix} + \begin{pmatrix} 4x_3 \\ -x_3 \\ 6x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

lineært system

$$\begin{aligned} x_1 + 3x_2 + 4x_3 &= 3 \\ -x_2 - x_3 &= 1 \\ 4x_1 + 2x_2 + 6x_3 &= 2 \end{aligned}$$

løses vha Gauss.

$$\left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & -1 & -1 & 1 \\ 4 & 2 & 6 & 2 \end{array}\right)$$

$\underline{v}_1$ $\underline{v}_2$ $\underline{v}_3$

utvidet matrise

Del 2: (1) Lineært $n\times n$ system med koeff. matrise A $\Big\}$ — $|A|\neq 0$: én løsning; $|A|=0$: ingen eller uendelig mange løsninger

(2) Kramer's regel:

I tilfellet $|A|\neq 0$:

$$\boxed{x_1 = \frac{|A_1(\underline{b})|}{|A|} \quad \vdots \quad x_n = \frac{|A_n(\underline{b})|}{|A|}}$$

Oppgavearbeid 23:

1b)

$$A = \begin{pmatrix} 2 & a & -1 \\ -1 & 2 & a \\ a & -1 & 2 \end{pmatrix}$$

$$|A| = 2(4+a) - a(-2-a^2) - 1(1-2a)$$

$$= 8 + 2a + a(a^2+2) + 2a - 1$$

$$= a^3 + 6a + 7 = (a+1)(a^2-a+7)$$

$\boxed{\pm 1, \pm 7 \quad a=-1 \text{ er løsning}}$

$|A|=0$: $a=-1$ eller $a^2-a+7=0$

$$\cancel{a = \frac{1 \pm \sqrt{1-4\cdot 7}}{2}}$$

Konklusjon: $a\neq -1$: $|A|\neq 0 \Rightarrow$ én løsning

$a=-1$:

$$\left(\begin{array}{ccc|c} 2 & -1 & -1 & -6 \\ -1 & 2 & -1 & -3 \\ -1 & -1 & 2 & 9 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & -2 & -9 \\ -1 & 2 & -1 & -3 \\ -1 & -1 & 2 & 9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & -9 \\ 0 & 3 & -3 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

uendelig mange løsn. (z fri)

4. $A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$

$$|A| = a(a^2-1) - 1(a-1) + 1(1-a)$$
$$= a(a-1)(a+1) \underbrace{-2a+2}_{-2(a-1)} = (a-1)(a(a+1)-2)$$
$$= (a-1)(a^2+a-2) = (a-1)(a+2)(a-1)$$

$|A|=0$: $a=1$ eller $a=-2$

i) $a=1$: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & -3 \end{array}\right) \Big] {-1} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ & \ldots & & \end{array}\right)$ ingen løsn.

ii) $a=-2$: $\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \\ 1 & 1 & -2 & -3 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 2 & 3 \\ 1 & -2 & 1 & 2 \\ 1 & 1 & -2 & -3 \end{array}\right) \Big]1 \Big]1 \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 2 & 3 \\ 0 & -3 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$

uendelig mange løsn. (z fri)

$$-3y+3z=5 \Rightarrow \frac{-3y}{-3} = \frac{5-3z}{-3}$$
$$y = -5/3 + z$$

$$-x-y+2z=3 \Rightarrow -x = 3-2z+(-5/3+z)$$
$$\frac{-x}{-1} = \frac{4/3 - z}{-1} \qquad x = -4/3 + z$$

Løsn: $(x,y,z) = (z-4/3,\ z-5/3,\ z)$, z fri

iii) $a \neq 1, -2$: $|A| = (a-1)^2(a+2) \neq 0$ én løsning

Kramer's regel:

$$x = \frac{|A_1(b)|}{|A|} = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & a & 1 \\ -3 & 1 & a \end{vmatrix}}{(a-1)^2(a+2)} = \frac{(a^2-1)-(2a+3)+(2+3a)}{(a-1)^2(a+2)}$$

$$x = \frac{a^2+a-2}{(a-1)^2(a+2)} = \frac{\cancel{(a+2)}\cancel{(a-1)}}{(a-1)^{2}\cancel{(a+2)}} = \frac{1}{a-1}$$

Tilsvarende for y og z.

6. $\underline{v_1} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\underline{v_2} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ $\underline{v_3} = \begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\underline{b}$ lineærkombinasjon hvis likninga har minst én løsn. JA.

$$x_1 \underline{v_1} + x_2 \underline{v_2} + x_3 \underline{v_3} = \underline{b}$$

$$x_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 7 \\ -8 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ 2x_1 \\ -x_1 \end{pmatrix} + \begin{pmatrix} 3x_2 \\ x_2 \\ 4x_2 \end{pmatrix} + \begin{pmatrix} x_3 \\ 7x_3 \\ -8x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 2 \\ 2x_1 + x_2 + 7x_3 &= -1 \\ -x_1 + 4x_2 - 8x_3 &= 5 \end{aligned}$$

$$\begin{array}{cccc} \begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 1 & 7 & -1 \\ -1 & 4 & -8 & 5 \end{pmatrix} \\ \underline{v_1} \;\; \underline{v_2} \;\; \underline{v_3} \;\; \underline{b} \end{array} \quad -2, \; 1$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -5 & 5 & -5 \\ 0 & 7 & -7 & 7 \end{array}\right) :5 \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 7 & -7 & 7 \end{array}\right) \; 7$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

uendelig mange løsn.
z fri

$-y + z = -1 \Rightarrow y = z+1$

$x + 3y + z = 2 \Rightarrow x = 2 - z - 3(z+1) = -4z - 1$

$(x, y, z) = (-4z-1, z+1, z)$

z fri

$z = 0$: $(-1, 1, 0)$

$-\underline{v_1} + \underline{v_2} = \underline{b}$

a.

$$\underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \underline{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \\ 3 \end{pmatrix} \quad \underline{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 7 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & -1 & b \\ 1 & 4 & 1 & c \\ 1 & 3 & 7 & d \end{array}\right) \begin{matrix}]-1 \\ ]-1 \\ ]-1 \end{matrix} \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & -2 & b-a \\ 0 & 3 & 0 & c-a \\ 0 & 2 & 6 & d-a \end{array}\right) \begin{matrix}]-3 \\ ]-2 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & -2 & b-a \\ 0 & 0 & 6 & c-a-3(b-a) \\ 0 & 0 & 10 & d-a-2(b-a) \end{array}\right)]-10/6$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & \cdot \\ 0 & 1 & -2 & \cdot \\ 0 & 0 & 6 & \cdot \\ 0 & 0 & 0 & * \end{array}\right)$$

$$* = \frac{6}{6}\left(d-a-2(b-a)\right) - \frac{10}{6}\cdot\left(c-a-3(b-a)\right)$$

$$= \frac{1}{6}\left(6d - 6a - 12b + 12a - 10c + 10a + 30b - 30a\right)$$

$$* = \frac{1}{6}\left(-14a + 18b - 10c + 6d\right)$$

Konklusjon: Én løsning hvis $-14a - 18b - 10c + 6d = 0$

Ingen løsning ellers $\quad$ — " — $\neq 0$

$(a,b,c,d) = (0,0,1,1)$: $\quad -10\cdot 1 + 6 \cdot 1 = -4 \neq 0$

$\Rightarrow$ ingen løsning

Dvs: $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ ikke lin. komb.

10.

a) $60x + 75y + 320z = 400.000$

budsjett betingelse

pris for varene = tilsv. beløp.

b)

	A	B	C
1	20	5	30
2	40	-50	180
3	-20	25	-265

$$\boxed{\begin{aligned} R_1 &= 20x + 5y + 30z \\ R_2 &= 40x - 50y + 180z \\ R_3 &= -20x + 25y - 265z \\ 400.000 &= 60x + 75y + 320z \end{aligned}}$$

Gevinst

$$\boxed{\begin{aligned} R_1 &= 50.000 \\ R_2 &= 25.000 \\ R_3 &= -100.000 \end{aligned}}$$

$C = 400.000$

$$\left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 40 & -50 & 180 & R_2 \\ -20 & 25 & -265 & R_3 \\ 60 & 75 & 320 & C \end{array}\right) \begin{array}{l} -2, \ 1, \ -3 \end{array} \rightarrow \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 30 & -235 & R_3 + R_1 \\ 0 & 60 & 230 & C - 3R_1 \end{array}\right) \begin{array}{l} \frac{1}{2}, \ +1 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + R_1 + \frac{1}{2}(R_2 - 2R_1) \\ 0 & 0 & 350 & C - 3R_1 + R_2 - 2R_1 \end{array}\right) \begin{array}{l} 2 \end{array}$$

$$\left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + \frac{1}{2}R_2 \\ 0 & 0 & 0 & C - 3R_1 + R_2 - 2R_1 + 2(R_3 + \frac{1}{2}R_2) \end{array}\right)$$

etc.

$= -100' - 125' = -1125'$

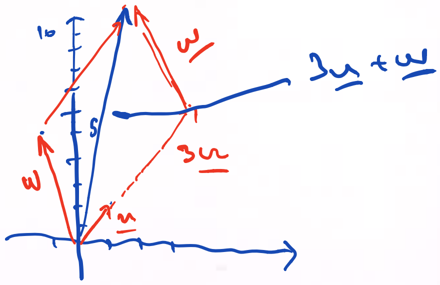
$$* \quad = C - 5R_1 + 2R_2 + 2R_3$$

Vi har løsning: $c - 5R_1 + 2R_2 + 2R_3 = 0$

$$\Updownarrow$$

$$c = 5R_1 - 2R_2 - 2R_3$$

$$400' = \underbrace{5\cdot 50' - 2\cdot 25' - 2\cdot(-100')}_{250' - 50' + 200'} = \underline{\underline{400'}} \quad \text{ok}$$



5. f) $3\underline{u} + \underline{w} = 3\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}-1\\5\end{pmatrix} = \begin{pmatrix}2\\11\end{pmatrix}$

7. $x_1\begin{pmatrix}1\\2\\-1\end{pmatrix} + x_2\begin{pmatrix}3\\1\\4\end{pmatrix} + x_3\begin{pmatrix}1\\7\\a\end{pmatrix} = \begin{pmatrix}2\\-1\\5\end{pmatrix}$

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 2 \\ 2x_1 + x_2 + 7x_3 &= -1 \\ -x_1 + 4x_2 + a\,x_3 &= 5 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 2 & 1 & 7 & -1 \\ -1 & 4 & a & 5 \end{array}\right) \begin{matrix} \\ -2 \\ 1 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -5 & 5 & -5 \\ 0 & 7 & a+1 & 7 \end{array}\right) :5 \to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 7 & a+1 & 7 \end{array}\right) \begin{matrix} \\ 7 \\ \end{matrix} \to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & a+8 & 0 \end{array}\right)$$

$\underline{a = -8}$: $\left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ uendelig mange løsninger, z fri

$-y + z = -1 \Rightarrow y = \underline{z+1}$

$x + 3y + z = 2 \Rightarrow x = 2 - z - 3(z+1) = \underline{-4z-1}$

$(x, y, z) = \underline{\underline{(-4z-1,\ z+1,\ z)}}$

z fri

$\underline{a \neq -8}$: én løsning

$$x + 3y + z = 2$$
$$-y + z = -1$$
$$(a+8)z = 0 \Rightarrow z = \frac{0}{a+8} = 0$$

$$x = -4z - 1 = -4 \cdot 0 - 1 = -1$$
$$y = z + 1 = 0 + 1 = 1$$

$$\Rightarrow (x, y, z) = \underline{\underline{(-1, 1, 0)}}$$

8.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & 4 & -1 \\ -1 & 1 & 1 & 5 \end{array}\right) \begin{matrix} -2 \\ \\ 1 \end{matrix} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & 2 & -5 \\ 0 & 2 & 2 & 7 \end{array}\right) \begin{matrix} \\ 2 \\ \end{matrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & 2 & -5 \\ 0 & 0 & 6 & -3 \end{array}\right)$$

én løsning

$$6z = -3 \Rightarrow z = -3/6 = \underline{-1/2}$$

$$-y + 2z = -5 \Rightarrow$$
$$-y = -5 - 2 \cdot (-1/2) = -4$$
$$\underline{y = 4}$$

$$x + y + z = 2$$
$$x = 2 - 4 - (-1/2) = \underline{-3/2}$$

$$(x, y, z) = \underline{\underline{(-3/2, 4, -1/2)}}$$