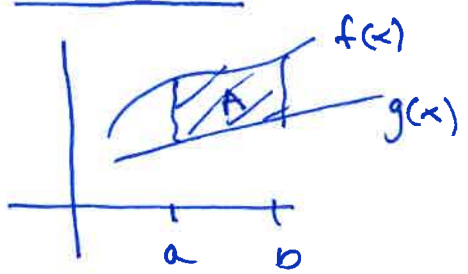
Plan

1. Lineære likningssystemer
2. Gauss-eliminasjon

Repetisjon:

i)

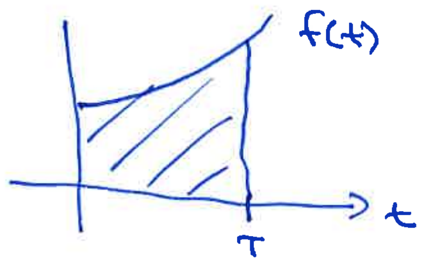


$$A = \int_a^b f(x) - g(x)\, dx$$

ii) Uegentlige integral

"$\pm\infty$" $\rightsquigarrow$ Gjør om til grenseverdi

iii)



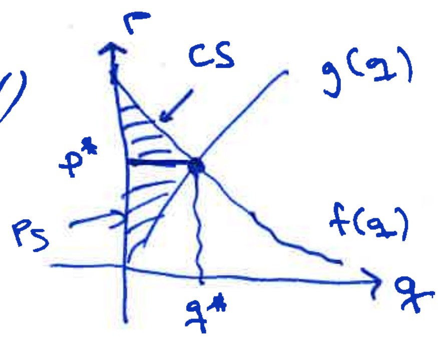
f(t) ← kont. kontantstrøm

Samlet kontantstrøm: $\int_0^T f(t)\, dt$

Samlet nåverdi: $\int_0^T f(t) \cdot e^{-rt}\, dt$

(r = disk. rente)

iv)



f(q) : (omvendt) ettersp. fn.
g(q) : " tilbuds fn.

$$CS = \int_0^{q^*} f(q) - p^*\, dq$$

$$PS = \int_0^{q^*} p^* - g(q)\, dq$$

Oppgave sett 20:

5 d) $\int_1^{\infty} \frac{1}{x^2+x}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x^2+x}\,dx$

i) $\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x} - \frac{1}{x+1}\,dx = \ln|x| - \ln|x+1| + C$

$= \underline{\ln \frac{|x|}{|x+1|} + C}$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \quad | \cdot \text{FN}$$

$$1 = A(x+1) + Bx \qquad A = 1$$

$$1 = \underbrace{(A+B)}_{=0}x + \underbrace{(A)}_{=1} \qquad B = -1$$

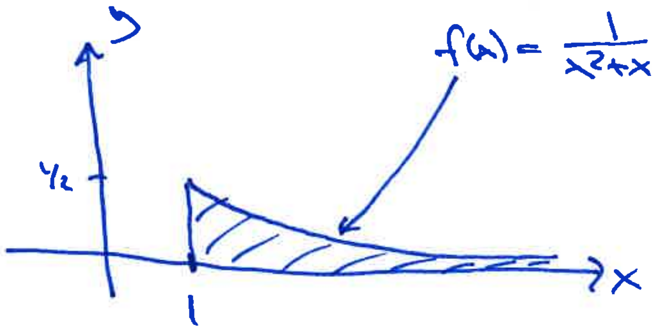
$$\ln a + \ln b = \ln(ab)$$
$$\ln a - \ln b = \ln(a/b)$$

ii) $\int_1^{b} \frac{1}{x^2+x}\,dx = \left[\ln\left|\frac{x}{x+1}\right|\right]_1^b = \ln\left(\frac{b}{b+1}\right) - \ln\left(\frac{1}{2}\right)$

iii) $\int_1^{\infty} \frac{1}{x^2+x}\,dx = \lim_{b\to\infty}\left(\ln \underbrace{\frac{b}{b+1}}_{\to 1} - \ln\frac{1}{2}\right) = -\ln\left(\frac{1}{2}\right) = \cancel{\ln 2}$

$= -\ln(2^{-1}) = \underline{\underline{\ln 2}}$

$\simeq \underline{\underline{0.693}}$

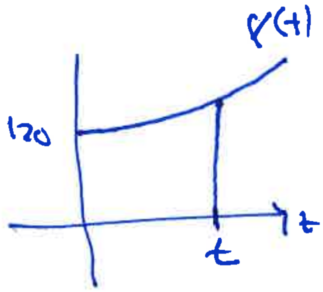


7. $V(t) = 120\, e^{\sqrt{t}/5}$ $\quad r = 4\%$

a) Maksimer nåverdi av salgssum:

$$N(t) = \frac{120\, e^{\sqrt{t}/5}}{e^{0.04t}}$$

nåverdien om vi selger på tidspunkt t.

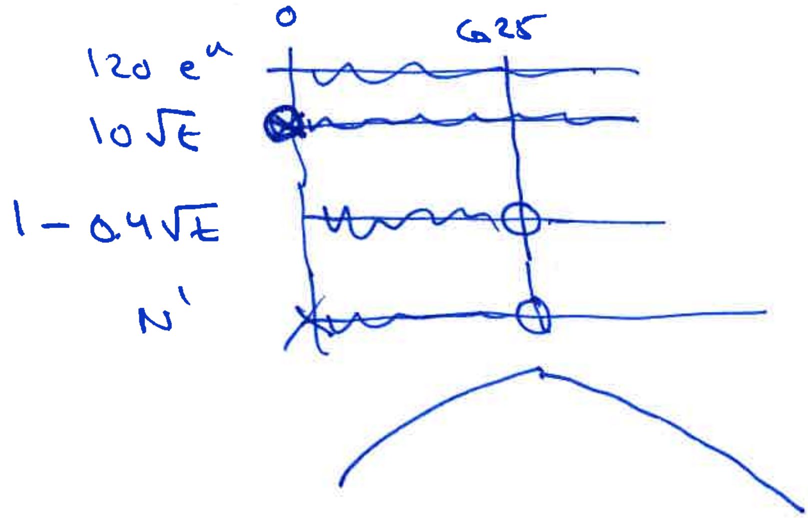


$$= 120\, e^{\sqrt{t}/5 - 0.04t}$$

$$= 120\, e^{u}, \quad u = \sqrt{t}/5 - 0.04t$$

$$N'(t) = 120\, e^{u} \cdot u' = 120\, e^{u} \cdot \left(\frac{1}{5} \cdot \frac{1}{2\sqrt{t}} - 0.04\right)$$

$$= 120\, e^{u} \cdot \frac{1 - 0.04 \cdot 10\sqrt{t}}{10\sqrt{t}}$$



$N' = 0$

$1 - 0.4\sqrt{t} = 0$

$1 = 0.4\sqrt{t}$

$\sqrt{t} = 1/0.4 = 2.5$

$t = 2.5^2 = \underline{6.25}$

maks: $\underline{\underline{t = 6.25}}$

b) T: tida før verdien er doblet

$V(t) = 240$

$120\, e^{\sqrt{t}/5} = 240$

$e^{\sqrt{t}/5} = 2$

$\sqrt{t}/5 = \ln 2$

$\sqrt{t} = 5 \ln 2$

$t = 5^2 (\ln 2)^2$

$= \underline{\underline{25 (\ln 2)^2}}$

Vis at: Etter $3T$ år til er verdien doblet på nytt

Dvs: $V(4T) = 480$

$V(t) = 480$

$120\, e^{\sqrt{t}/5} = 480$

$e^{\sqrt{t}/5} = 4$

$\sqrt{t}/5 = \ln 4 = 2 \ln 2$

$\sqrt{t} = 5 \cdot 2 \ln 2$

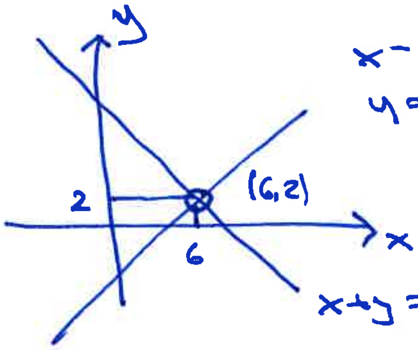
$t = \underline{\underline{100 (\ln 2)^2}} = \underline{\underline{4T}}$

① Lineære likningssystem

Eks.

$$x+y=8$$
$$x-y=4$$

lineært



$x-y=4$
$y=x-4$

$x+y=8$
$y=8-x$

Løsning: $(x,y)=(6,2)$

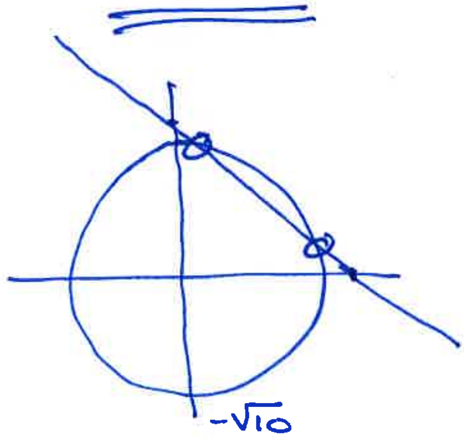
‖

Skjæringspunkt

Løsning: Tallpar (x,y) som tilfredsstiller begge likn. samtidig

$$x^2+y^2=10 \quad 2$$
$$x+y=4 \quad 1$$

ikke lineær



Løsn: Innsetting

$$y=4-x$$
$$x^2+(4-x)^2=10$$
$$x^2+16-8x+x^2=10$$
$$2x^2-8x+6=0$$
$$x=1, \quad x=3$$
$$y=3 \quad y=1$$

Løsn:
$(x,y)=(1,3),(3,1)$

Eks:

$$\boxed{\begin{aligned} x+y+z&=3\\ x+2y+4z&=7\\ x+3y+9z&=13 \end{aligned}}$$

3×3 lineært system

m×n lineært system:

- m likninger
- n variabler
- alle likninger er lineære

Eliminasjon:

(1) $x+y+z=3$
(2) $x+2y+4z=7$
(3) $x+3y+9z=13$

(a) $x+y+z=3$ (1)
(b) $y+3z=4$ (2) − (1)
(c) $2y+8z=10$ (3) − (1)

$x+y+z=3$ (a)
$y+3z=4$ (b)
$\boxed{2z=2}$ (c) − 2(b)

Baklengs substitusjon:

$2z=2 \Rightarrow z=1$
$y+3z=4 \Rightarrow y=4-3=1$
$x+y+z=3 \Rightarrow x=3-1-1=1$

$(x,y,z)=(1,1,1)$

Substitusjon / innsetting:

$x=3-y-z$ $x=1$

$\boxed{\begin{aligned} y+3z&=4\\ 2y+8z&=10 \end{aligned}}$ $\leftarrow (3-y-z)+2y+4z=7$, $\leftarrow (3-y-z)+3y+9z=13$

2×2

$y=4-3z$ $y=1$

$2(4-3z)+8z=10$

$\boxed{2z=2}$ $z=1$

1×1

Løsning: $(x,y,z)=(1,1,1)$

② Gauss-eliminasjon : generell, rask og systematisk metode for å løse lineære system

Defn:

Et $m \times n$ lineært system har formen:

m likninger, n ukjente $x_1, x_2, \ldots, x_n$

$$m \left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n} \cdot x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn} \cdot x_n = b_m \end{array} \right.$$

n ukjente $(x_1, x_2, \ldots, x_n)$

der $a_{11}, a_{12}, \ldots, a_{mn}, b_1, \ldots, b_m$ er gitte tall.

med matriser:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \text{og} \quad (A|b) = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

(koeffisientmatrisen) (utvidet matrise)

Eks:
$$\begin{array}{l} x + y + z = 3 \\ x + 2y + 4z = 7 \\ x + 3y + 9z = 13 \end{array} \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 7 \\ 1 & 3 & 9 & 13 \end{array}\right)$$

Gauss-eliminasjon:

lineært system $\rightsquigarrow$ utvidet matrise

$\Big\}$ elementære radoperasjoner

lineært system $\leftarrow$ trappeform

$\Big\}$

Baklengs substitusjon

$\Big\}$

Løsninger

Eks:

$$x+y+z=4$$
$$x-y+2z=3$$
$$x+2y+3z=7$$

$$\downarrow$$

vil få null

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 7 \end{array}\right) \cdot(-1)$$

$$\downarrow$$

(2)−(1)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ 1 & 2 & 3 & 7 \end{array}\right) \cdot(-1)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ 0 & 1 & 2 & 3 \end{array}\right) \cdot 2$$

vil få null

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ 0 & 2 & 4 & 6 \end{array}\right) \cdot 1$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 5 & 5 \end{array}\right)$$

trappeform

$$x+y+z=4$$
$$-2y+z=-1$$
$$5z=5$$

$x=2$, $y=1$, $z=1$

Elementære radoperasjoner:

i) bytte om to rader

ii) multiplisere en rad med $c \neq 0$

iii) legge til et multippel av en rad til en annen rad

Pivot: første tall i en rad ulik null

Trappeform: En matrise slik at

i) alle tall under en pivot er null

ii) alle null-rader står nederst i matrisen

Løsning:

$(x,y,z) = (2,1,1)$

Teorem:

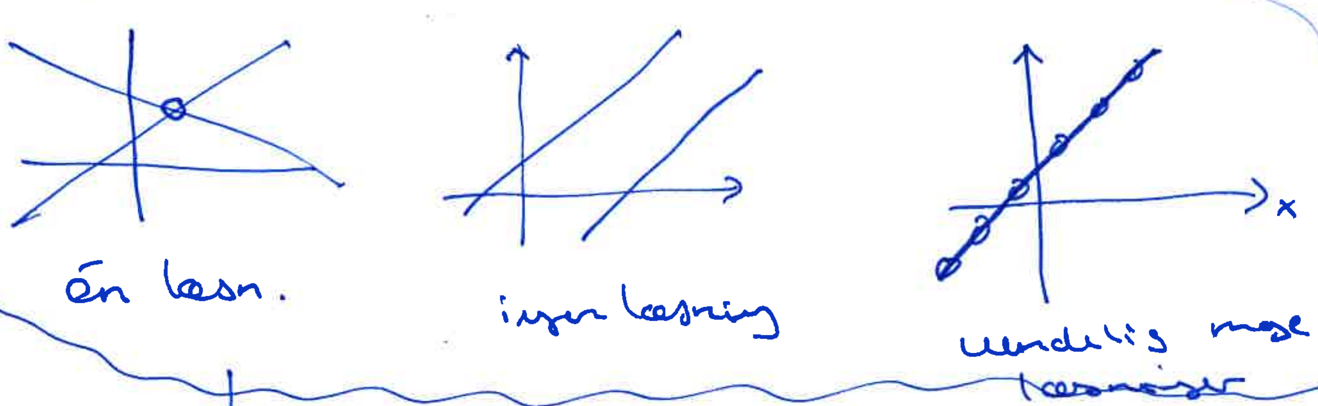
Et $m \times n$ lineært system har enten

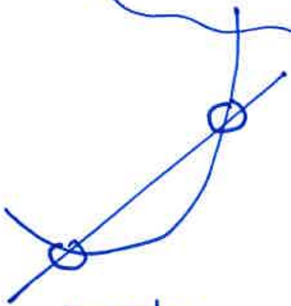
i) én løsning

ii) ingen løsninger

iii) uendelig mange løsninger

Eks: 2×2





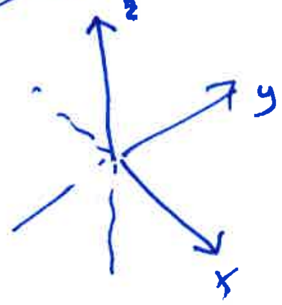
umulig for lineære system

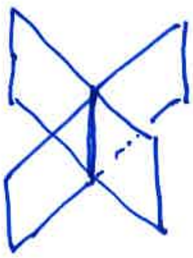
Eks: 3×3

$$x + y + z = 1 \Rightarrow z = 1 - x - y$$

$$z = f(x, y)$$

grafen til en lineær funksjon = et plan





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Eks:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

trappeform

$x + y + z = 4$
$2y + z = 6$
$0 = 3$
umulig
$\Downarrow$
ingen løsning.

Ingen løsning $\Longleftrightarrow$ Trappeformen har pivot i siste kolonne

Eks:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

trappeform

$x + y + z = 4$
$2y + z = 6$
~~$0 = 0$~~

Baklengs substitusjon:

$$\frac{2y}{2} = \frac{6 - z}{2}$$
$$y = 3 - \tfrac{1}{2}z$$

$x + y + z = 4$
$x = 4 - y - z$
$= 4 - (3 - \tfrac{1}{2}z) - z$
$x = 1 - \tfrac{1}{2}z$

$$(x, y, z) = (1 - \tfrac{1}{2}z,\ 3 - \tfrac{1}{2}z,\ z)$$

der z er fri variabel

uendelig mange løsninger,
én fri variabel / en frihetsgrad

frie variable $\Longleftrightarrow$ kolonner i trappeformen uten pivot

Gauss-eliminasjon:

i) Vi kan alltid komme frem til en trappeform

ii) En trappeform er ikke entydig, men posisjonen til pivotene i trappeformen er entydig.