Key Problems

Problem 1.
We consider the constrained optimization problem max $f(x,y,z) = 2x^2 - 4y^2 - 2z^2$ when $x^4 + y^4 + z^4 \leq 16$.

a) Find the maximum point and maximum value of f.

b) Use the envelope theorem to estimate the new maximum value of f when we
i) change the constraint to $x^4 + y^4 + z^4 \leq 20$,
ii) change the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$,
iii) change the constraint to $x^4 + y^4 + z^4 \leq 20$ and the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$

Problem 2.
Determine the range of the following quadratic functions:

a) $f(x,y,z) = x^2 + 4xz + y^2 + 5z^2 - 4y + 2z$

b) $f(x,y,z,w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 6z^2$

c) $f(x,y,z,w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 6z^2 + 3w^2 + 1$

Problem 3.
We consider the Lagrange problem given by

$$\min\ f(x,y,z,w) = -4x^2 - 10y^2 - 5z^2 - 5w^2 + 4xz + 4xw - 4yz + 4yw + 6zw \text{ when } x^2 + y^2 + z^2 + w^2 = 6$$

a) Determine whether f is convex or concave.

b) Find all points (x,y,z,w) such that $(x,y,z,w;\lambda)$ satisfy the Lagrange conditions when $\lambda = -12$.

c) Show that any point satisfying the conditions in b) is a minimum point.

d) Solve max $f(x,y,z,w)$ subject to $x^2 + y^2 + z^2 + w^2 = 6$.

Problem 4.
We consider the returns of the securities Apple and Bakkafrost. Based on marked data from May 2010 to September 2021, the mean monthly continuous returns and their covariance are given by

$$\mu = \begin{pmatrix} 0.0210 \\ 0.0235 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 0.0058 & 0.0004 \\ 0.0004 & 0.0070 \end{pmatrix}$$

Find the minimum variance portfolio, its expected return and standard deviation.

Exercise Problems

Exam problems [Final exam] 01/2018 Q1,3,4,5

Answers to Key Problems

Problem 1.

a) $(x,y,z;\lambda) = (\pm 2,0,0;1/4)$ with $f(\pm 2,0,0) = 8$

b) *i)* $f_{\max} \cong 9$ *ii)* $f_{\max} \cong 4$ *iii)* $f_{\max} \cong 5$

Problem 2.

a) $[-5,\infty)$

b) $(-\infty,\infty)$

c) $[1,\infty)$

Problem 3.

a) f is concave

b) $(0,-2,-1,1;-12), (0,2,1,-1;-12)$

c) Using SOC

d) $f_{\max} = 0$

Problem 4.

$\omega_M = (0.55, 0.45)$, $\mu_M = 0.0221$, $\sigma_M = 0.0581$