Exam	Mock midterm exam in GRA 6035 Mathematics
Date	October 12th, 2023 at 1700-1800

This exam consists of 8 problems with score 0 - 3p each, and maximal score on this exam is 24p.
You must give reasons for your answers.

Question 1.
Determine the rank of the matrix A:

$$A = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 2 & 3 & -1 & 3 \\ 3 & 4 & 0 & 2 \end{pmatrix}$$

Question 2.
In how many ways is it possible to write $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix},$$

Question 3.
Determine the equilibrium state of the Markov chain with transition matrix A:

$$A = \begin{pmatrix} 0.79 & 0.07 \\ 0.21 & 0.93 \end{pmatrix}$$

Question 4.
Determine the definiteness of the quadratic form $q(x, y, z) = 2x^2 + 8xy + 12xz + 9y^2 + 26yz + 20z^2$.

Question 5.
The points $(1, 7, 3, 5)$ and $(0, 3, 0, 12)$ are solutions of a 4×4 linear system $A\mathbf{x} = \mathbf{b}$. Determine the value of the determinant of A.

Question 6.
Determine the dimension of the eigenspace E_λ of A which contains the vector $\mathbf{v}$:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Question 7.
Solve the optimization problem: $\min f(x, y, z) = x^2 + y^2 + z^2 + xy + yz - 2x - 2y - 2z + 7$.

Question 8.
Determine whether the matrix A is diagonalizable:

$$A = \begin{pmatrix} 0 & 1 & 5 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$