Exam	Mock Midterm exam
Date	October 2022

About the midterm exam

The midterm exam is a 1h pen and paper exam. It will consist of 8 problems with score 0 - 3p each. The exercise questions below are of a similar type as the midterm exam questions. Many of them are copied from previous midterm exams (reformulated to not be multiple choice questions).

On the midterm exam, you must give reasons for your answers. Short and precise answers will be rewarded. A short computation and/or reference to relevant theory, given in a couple of lines, is enough.

Exercise problems

Question 1.
Determine the dimension of the null space of the matrix A:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Question 2.
The points $(x, y, z) = (2, 4, 0)$ and $(x, y, z) = (3, 7, 2)$ are solutions of a linear system. Are there any solutions of the linear system with $x = y = 1$?

Question 3.
Determine the equilibrium state of the Markov chain with transition matrix A:

$$A = \begin{pmatrix} 0.74 & 0.13 \\ 0.26 & 0.87 \end{pmatrix}$$

Question 4.
Determine whether the vector $\mathbf{v}$ is in the column space of the matrix A:

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 5 & 4 & 6 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

Question 5.
Compute the orthogonal projection $\text{proj}_{\mathbf{w}}(\mathbf{v})$ when $\mathbf{v} = (1, 2, 0, 1)$ and $\mathbf{w} = (1, 1, 1, 1)$.

Question 6.
Let A be a 4×5 matrix with minors given by $M_{1234,1234} = 0$, $M_{123,123} \neq 0$, and $M_{1234,1235} = 0$. Determine the dimension of $\text{Null}(A)$.

Question 7.
Determine all values of t such that the vectors are linearly independent:

$$\mathbf{v}_1 = \begin{pmatrix} t \\ 2 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 6 \\ t \\ 9+t \end{pmatrix}$$

Question 8.
Find the characteristic equation of the matrix A:

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 5 \\ 3 & 5 & 8 \end{pmatrix}$$

Question 9.
Determine the equilibrium state of the Markov chain with transition matrix A:

$$A = \begin{pmatrix} 0.4 & 0.2 & 0.2 \\ 0.4 & 0.6 & 0.1 \\ 0.2 & 0.2 & 0.7 \end{pmatrix}$$

Question 10.
Determine all values of s such that the matrix A diagonalizable:

$$A = \begin{pmatrix} 3 & 0 & s \\ 0 & s & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Question 11.
Determine the rank of A when A is a 3×3 matrix and the eigenspace E_0 of A has base $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1 = (-1, 1, 0)$ and $\mathbf{v}_2 = (2, 0, 1)$.

Question 12.
Determine the definiteness of the quadratic form q:

$$q(x, y, z) = 2x^2 + 2xy + 6xz + 4y^2 + 10yz + 8z^2$$

Question 13.
Determine the rank of the matrix A:

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

Question 14.
The subset V of $\mathbb{R}^5$ is the set of all solutions of the linear system

$$\begin{pmatrix} 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & -1 & 1 & 0 \end{pmatrix} \cdot \mathbf{x} = \mathbf{0}$$

that also satisfies $x_1 + x_2 + x_3 + x_4 + x_5 = 0$. Determine the dimension of V.

Question 15.
Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & t & t^2 & t^3 \\ 1 & 1 & t & t^2 \end{pmatrix}$$

Question 16.
Find the eigenvalues of the matrix A, and determine their multiplicities:

$$A = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 5 \end{pmatrix}$$

Question 17.
Show that $\lambda = -1$ is an eigenvalue of the matrix A, and determine its multiplicity:

$$A = \begin{pmatrix} 2 & 3 & 3 & 3 \\ 3 & 2 & 3 & 3 \\ 3 & 3 & 2 & 3 \\ 3 & 3 & 3 & 2 \end{pmatrix}$$

Question 18.
Determine the definiteness of the quadratic form q:

$$q(x, y, z, w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 6z^2$$

Question 19.
Find an orthonormal base of $\text{Null}(A + 2I)$ when A is the matrix

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$

Question 20.
Let A be a symmetric 3×3 matrix such that $\det(A) = -6$ and $\text{tr}(A) = 4$. Determine its definiteness.

Question 21.
Consider the function $f(x, y, z) = x^4 + y^4 + z^4 - 4xyz$. Show that $(x, y, z) = (1, 1, 1)$ is a stationary points of f and classify its type.

Question 22.
The function $g(x, y, z, w)$ is concave and a is a parameter. Determine whether $f(x, y, z, w) = 3 - a \cdot g(x, y, z, w)$ is convex or concave.