Exam	Midterm exam in GRA 6035 Mathematics
Date	January 9th, 2023 at 1700 - 1800

This exam consists of 8 problems with score 0 - 3p each, and maximal score on this exam is 24p.
You must give reasons for your answers.

Question 1.
Determine the dimension of the column space of the matrix A:

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 3 & 5 & 4 \end{pmatrix}$$

Question 2.
Write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

Question 3.
Determine all values of s such that the matrix A has maximal rank:

$$A = \begin{pmatrix} 3 & s & 1 \\ 1 & 1 & s \\ 4 & 3 & 3 \end{pmatrix}$$

Question 4.
Determine the equilibrium state of the Markov chain with transition matrix A:

$$A = \begin{pmatrix} 0.58 & 0.06 \\ 0.42 & 0.94 \end{pmatrix}$$

Question 5.
Determine the definiteness of the quadratic form $q(x, y, z) = -x^2 + 4xy + 2xz - 3y^2 - 4yz - z^2$.

Question 6.
Determine λ such that the vector $\mathbf{v}$ is in the eigenspace E_λ of A:

$$A = \begin{pmatrix} 2 & 3 & -1 & 0 \\ 3 & 2 & 0 & -2 \\ -1 & 0 & 3 & 1 \\ 0 & -2 & 1 & 3 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}$$

Question 7.
Determine whether the function $f(x, y, z) = x^4 + 2x^2 + 3y^2 - 6xz + 6z^2$ is convex or concave.

Question 8.
Find two linearly independent vectors that are orthogonal to $(1, 3, 2)$.