Exam	Midterm exam in GRA 6035 Mathematics
Date	October 14th, 2022 at 0900 - 1000

This exam consists of 8 problems with score 0 - 3p each, and maximal score on this exam is 24p.
You must give reasons for your answers.

Question 1.
Determine the dimension of the null space of the matrix A:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \end{pmatrix}$$

Question 2.
Write $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix},$$

Question 3.
Determine all values of t such that the matrix A has maximal rank:

$$A = \begin{pmatrix} 1 & 1 & t \\ t & 3 & 1 \\ 3 & 4 & 3 \end{pmatrix}$$

Question 4.
Determine the equilibrium state of the Markov chain with transition matrix A:

$$A = \begin{pmatrix} 0.72 & 0.07 \\ 0.28 & 0.93 \end{pmatrix}$$

Question 5.
Determine the definiteness of the quadratic form $q(x, y, z) = -x^2 + 4xy + 2xz - 4y^2 - 4yz - z^2$.

Question 6.
Determine the dimension of the eigenspace E_λ of A which contains the vector $\mathbf{v}$:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Question 7.
Determine the scalar a such that $\mathbf{v} - a \cdot \mathbf{w}$ is orthogonal to $\mathbf{w}$ when $\mathbf{v} = (1, 0, 4, 3)$ and $\mathbf{w} = (1, 1, 1, 7)$.

Question 8.
The points $(1, 1, 3, 4)$ and $(0, 3, 1, 2)$ are solutions of a 3×4 linear system $A\mathbf{x} = \mathbf{b}$, where the minor $M_{123,124} = 2$. Determine all solutions of the linear system of the form $(x, y, z, 0)$.