

# GRA 60352

## Mathematics

Department of Economics

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**Finish date:** 11.10.2019 Time 16.00

**Weight:** 20% of GRA 6035

**Total no. of pages:** 5 incl. front page

**Answer sheets:** Answer sheets for multiple-choice examinations

**Examination support materials permitted:** BI-approved exam calculator. Simple calculator. Bilingual dictionary.

**Question 1.**

Consider the linear system with augmented matrix

$$\left( \begin{array}{cccc|c} 1 & 4 & 3 & 5 & 7 \\ 0 & 0 & 0 & 5 & 13 \\ 0 & 4 & 0 & 1 & 5 \end{array} \right)$$

**Which statement is true?**

- a) The linear system has a unique solution
- b) The linear system is inconsistent
- c) The linear system has one degree of freedom
- d) The linear system has two degrees of freedom
- e) I prefer not to answer.

**Question 2.**

Let the vectors  $\mathbf{v}_1$ ,  $\mathbf{v}_2$ ,  $\mathbf{v}_3$  and  $\mathbf{v}_4$  be the column vectors of the matrix

$$A = \begin{pmatrix} 1 & 4 & 3 & 5 \\ 0 & 0 & 0 & 5 \\ 0 & 4 & 0 & 1 \end{pmatrix}$$

**Which statement is true?**

- a) The vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$  are linearly independent
- b) The vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  are linearly independent and  $\text{rk } A = 3$
- c) The vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$  are linearly independent and  $\text{rk } A = 3$
- d) There are not three linearly independent vectors among  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$
- e) I prefer not to answer.

**Question 3.**

Consider the matrix

$$A = \begin{pmatrix} 7 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 7 \end{pmatrix}$$

**Which statement is true?**

- a)  $A$  has three distinct eigenvalues
- b)  $A$  has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
- c)  $A$  has an eigenvalue of multiplicity three
- d)  $A$  has one eigenvalues of multiplicity one, and no other eigenvalues
- e) I prefer not to answer.

**Question 4.**

Consider the matrix

$$A = \begin{pmatrix} t & -t & 4 & 1 \\ -1 & 6 & 3 & t \end{pmatrix}$$

**Which statement is true?**

- a) For  $t = 0$ , we have that  $\text{rk}(A) = 1$ , otherwise  $\text{rk}(A) = 2$
- b) For  $t = 0$  and  $t = 8$ , we have that  $\text{rk}(A) = 1$ , otherwise  $\text{rk}(A) = 2$
- c) For  $t = 0$ ,  $t = 8$  and  $t = 3/4$ , we have that  $\text{rk}(A) = 1$ , otherwise  $\text{rk}(A) = 2$
- d) For all values of  $t$ , we have that  $\text{rk}(A) = 2$
- e) I prefer not to answer.

**Question 5.**

A Markov chain  $\mathbf{x}_{t+1} = A \cdot \mathbf{x}_t$  has transition matrix  $A$  and equilibrium state  $\mathbf{v}$  given by

$$A = \begin{pmatrix} 0.40 & 0.20 & 0.10 \\ 0.40 & 0.60 & 0.10 \\ 0.20 & 0.20 & 0.80 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

**Which statement is true?**

- a)  $v_1 < 0.20$
- b)  $v_1 = 0.20$
- c)  $0.20 < v_1 < 0.25$
- d)  $v_1 \geq 0.25$
- e) I prefer not to answer.

**Question 6.**

Consider the quadratic form

$$f(x, y, z) = 2xy - x^2 - 2y^2 + 2yz - z^2$$

**Which statement is true?**

- a)  $f$  is indefinite
- b)  $f$  is negative semi-definite but not negative definite
- c)  $f$  is positive semi-definite but not positive definite
- d)  $f$  is negative definite
- e) I prefer not to answer.

**Question 7.**

Consider the function  $f(x, y, z) = x^3 + y^3 + z^3 - 3xz$ . **Which statement is true?**

- a) All stationary points of  $f$  are saddle points
- b) The function  $f$  has both a saddle point and a local minimum point.
- c) The function  $f$  has a local minimum point, and  $f$  is convex.
- d) The function  $f$  has a global minimum point, and  $f$  is convex.
- e) I prefer not to answer.

**Question 8.**

Let  $A$  be a  $3 \times 3$  matrix, such that  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$  is a base for  $\text{Null}(A)$  with

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

**Which statement is true?**

- a) The only solutions of the linear system  $A\mathbf{x} = \mathbf{0}$  are  $(1, 0, 1)$  and  $(0, 1, 4)$ .
- b)  $\text{rk} A = 2$
- c) All solutions of  $A\mathbf{x} = \mathbf{0}$  are given by  $(1, t, 1 + 4t)$ , where  $t$  is a free variable
- d) The linear system  $A\mathbf{x} = \mathbf{0}$  has two degrees of freedom

# SVARARK TIL FLERVALGSEKSAMEN

## ANSWER SHEET FOR MULTIPLE CHOICE EXAMINATION

Eksamenskode:  
Examination code:

G R A 6 0 3 5 2

Initialer:  
Personal initials:

NN

ID-nummer: (SKAL fylles ut!)  
ID-number: (MUST be filled in!)

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Skriv tydelig!  
Fyll ut med   
Annuler kryss med   
Helt fylt rute blir ikke registrert

Write clearly!  
Record answer with   
Cancel a cross with   
Compl. filled boxes will not be registered

Dette svararket leses kun av en maskin. Ikke noe av det du skriver utenom de definerte feltene blir lest eller tatt hensyn til.  
Ikke kluss på arket. Be heller om et nytt.

This answer sheet is only read by a machine. Answers or comments written on the examination paper or outside the boxes will not be graded.  
Do not scribble on this sheet.  
Please ask for a new answer sheet if you need one.

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