Exam	Midterm exam in GRA 6035 Mathematics
Date	January 6th, 2017 at 0900 - 1000

This exam has 8 questions

Question 1.

Consider an 5×6 linear system $A \cdot \mathbf{x} = \mathbf{b}$, where $\operatorname{rk} A = 5$. **Which statement is true?**

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has more than one degree of freedom
(e) I prefer not to answer.

Question 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$\mathbf{v}_1 = \begin{pmatrix} 4 \\ -4 \\ 2s \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} s \\ 3 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all s
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $s = 6$
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $s = 3$ or $s = 6$
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $s = 1$ or $s = 6$
(e) I prefer not to answer.

Question 3.

Compute the number of degrees of freedom of the linear system $A \cdot \mathbf{x} = \mathbf{0}$ when

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 3 & 1 & 4 & -1 \\ 1 & s & s+1 & 9 \end{pmatrix}$$

Which statement is true?

(a) There are 2 degrees of freedom for all s
(b) There are 2 degrees of freedom when $s = 3$, and 1 degrees of freedom when $s \neq 3$
(c) There are 2 degrees of freedom when $s = 7$, and 1 degrees of freedom when $s \neq 7$
(d) There is 1 degree of freedom for all s
(e) I prefer not to answer.

Question 4.

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of the matrix

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

Which statement is true?

(a) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1\lambda_2\lambda_3 > 0$
(b) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1\lambda_2\lambda_3 < 0$
(c) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1\lambda_2\lambda_3 > 0$
(d) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1\lambda_2\lambda_3 < 0$
(e) I prefer not to answer.

Question 5.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & s \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s = 1$
(c) A is diagonalizable exactly when $s \neq 1$
(d) A is not diagonalizable for any s
(e) I prefer not to answer.

Question 6.

A Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$ has transition matrix A given by

$$A = \begin{pmatrix} 0.71 & 0.29 \\ 0.29 & 0.71 \end{pmatrix}$$

and equilibrium state $\mathbf{x} = \binom{x}{y}$. **Which statement is true?**

(a) $x = 29$ and $y = 71$
(b) $x = 1$ and $y = 1$
(c) $x = 29/71$ and $y = 42/71$
(d) $x = 1/2$ and $y = 1/2$
(e) I prefer not to answer.

Question 7.

Consider the quadratic form

$$f(x, y, z) = 5x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2$$

Which statement is true?

(a) f is positive semidefinite but not positive definite
(b) f is positive definite but not positive semidefinite
(c) f is indefinite
(d) f is negative semidefinite
(e) I prefer not to answer.

Question 8.

Consider the function $f(x, y) = x/y + y/x$ defined on $D_f = \{(x, y) : x > 0, y > 0\}$. **Which statement is true?**

(a) f is neither convex nor concave
(b) f is convex but not concave
(c) f is concave but not convex
(d) f is both convex and concave
(e) I prefer not to answer.