

This exam has 8 questions

QUESTION 1.

Consider an $m \times n$ linear system $A \cdot \mathbf{x} = \mathbf{b}$, where $\text{rk } A = 4$ and $\text{rk}(A|\mathbf{b}) = 5$. **Which statement is true?**

- (a) The linear system has a unique solution
- (b) The linear system is inconsistent
- (c) The linear system has one degree of freedom
- (d) The linear system has $n - 4$ degrees of freedom
- (e) I prefer not to answer.

QUESTION 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$\mathbf{v}_1 = \begin{pmatrix} 4 \\ t \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2t \\ 0 \\ 1 \end{pmatrix}$$

Which statement is true?

- (a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all t
- (b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 1$
- (c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 3$ or $t = 6$
- (d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 1$ or $t = 6$
- (e) I prefer not to answer.

QUESTION 3.

Compute the number of degrees of freedom of the linear system $A \cdot \mathbf{x} = \mathbf{0}$ when

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & -1 & 1 & -3 \\ 1 & s & s+1 & 9 \end{pmatrix}$$

Which statement is true?

- (a) There are 2 degrees of freedom for all s
- (b) There are 2 degrees of freedom when $s = 3$, and 1 degrees of freedom when $s \neq 3$
- (c) There are 2 degrees of freedom when $s = 7$, and 1 degrees of freedom when $s \neq 7$
- (d) There is 1 degree of freedom for all s
- (e) I prefer not to answer.

QUESTION 4.

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1/4 & 0 & 1 \end{pmatrix}$$

Which statement is true?

- (a) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1\lambda_2\lambda_3 > 0$
- (b) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1\lambda_2\lambda_3 < 0$
- (c) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1\lambda_2\lambda_3 > 0$
- (d) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1\lambda_2\lambda_3 < 0$
- (e) I prefer not to answer.

QUESTION 5.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{pmatrix}$$

Which statement is true?

- (a) A is diagonalizable for all s
- (b) A is diagonalizable exactly when $s = 1$
- (c) A is diagonalizable exactly when $s \neq 1$
- (d) A is not diagonalizable for any s
- (e) I prefer not to answer.

QUESTION 6.

A Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$ has transition matrix A given by

$$A = \begin{pmatrix} 0.77 & 0.19 \\ 0.23 & 0.81 \end{pmatrix}$$

and equilibrium state $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$. Which statement is true?

- (a) $x = 19$ and $y = 23$
- (b) $x = 19/23$ and $y = 4/23$
- (c) $x = 23/42$ and $y = 19/42$
- (d) $x = 19/42$ and $y = 23/42$
- (e) I prefer not to answer.

QUESTION 7.

Consider the quadratic form

$$f(x_1, x_2, x_3, x_4) = 2x_1^2 + 6x_1x_4 + 4x_2^2 + 2x_2x_3 - 2x_2x_4 + 3x_3^2 + 5x_4^2$$

Which statement is true?

- (a) f is positive semidefinite but not positive definite
- (b) f is positive definite
- (c) f is indefinite
- (d) f is negative definite
- (e) I prefer not to answer.

QUESTION 8.

For $a \neq 0$, consider the function $f(x, y) = x \ln y - ay \ln x$ defined on $D_f = \{(x, y) : x > 0, y > 0\}$.
Which statement is true?

- (a) f is a concave function for $a = 1$ and for $a = -1$
- (b) f is a concave function for $a = 1$, but not for $a = -1$
- (c) f is a concave function for $a = -1$, and not for $a = 1$
- (d) f is not a concave function for $a = 1$ or for $a = -1$
- (e) I prefer not to answer.

SVARARK TIL FLERVALGSEKSAMEN

ANSWER SHEET FOR MULTIPLE CHOICE EXAMINATION

Eksamenskode:
Examination code:

G R A 6 0 3 5 2

Initialer:
Personal initials:

NN

ID-nummer: (SKAL fylles ut!)
ID-number: (MUST be filled in!)

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Skriv tydelig!
Fyll ut med
Annuler kryss med
Helt fylt rute blir ikke registrert

Write clearly!
Record answer with
Cancel a cross with
Compl. filled boxes will not be registered

Dette svararket leses kun av en maskin. Ikke noe av det du skriver utenom de definerte feltene blir lest eller tatt hensyn til.
Ikke kluss på arket. Be heller om et nytt.

This answer sheet is only read by a machine. Answers or comments written on the examination paper or outside the boxes will not be graded.
Do not scribble on this sheet.
Please ask for a new answer sheet if you need one.

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