Solutions:	**GRA 60352**	**Mathematics**		
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			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Ordinary exam			Responsible department: Economics	

Correct answers: C-D-B-A-C-D-B-D

Question 1.

The linear system is consistent since it is homogeneous. It has $n - \text{rk}\, A = 3 - 2 = 1$ degrees of freedom. The correct answer is alternative **C**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 1 & 8 & 1 \\ 1 & a^2 & -1 \\ 1 & a^3 & 1 \end{vmatrix} = 2a^3 - 16 = 2(a^3 - 8)$$

This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $a \neq 2$, and linearly dependent if $a = 2$. The correct answer is alternative **D**.

Question 3.

We compute the minor of order 3 obtained from the first three columns:

$$\begin{vmatrix} h & 4 & 7 \\ 3 & 1 & 0 \\ -1 & 3 & 7 \end{vmatrix} = 7(h - 2)$$

It follows that the rank of A is 3 if $h \neq 2$. If $h = 2$, we see that the first row is equal to the sum of the two last rows, so the rank is at most two. Since column two and three are linearly independent, the rank is two. The correct answer is alternative **B**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 0 & 3-\lambda & 0 \\ 4 & 0 & 7-\lambda \end{vmatrix} = (3-\lambda)(\lambda^2 - 8\lambda + 11) = 0$$

Hence the eigenvalues of A are $\lambda = 3$ and $\lambda = 4 \pm \sqrt{5} > 0$. The correct answer is alternative **A**.

Question 5.

The eigenvalues are the numbers $1, 3, s$ on the diagonal since A is upper triangular. When $s \neq 1, 3$ the matrix has three distinct eigenvalues and is diagonalizable. If $s = 3$, then $\text{rk}(A - 3I) = 1$ so the linear system has two degrees of freedom. Since the multiplicity of $\lambda = 3$ is two, the matrix is diagonalizable. If $s = 1$, then $\text{rk}(A - I) = 2$ so the linear system has one degree of freedom. Since the multiplicity of $\lambda = 1$ is two, the matrix is not diagonalizable for $s = 1$. The correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $f(x_1, x_2, x_3, x_4) = -x_1^2 + 4x_1x_2 + 2x_1x_3 - 5x_2^2 - 6x_3^2 - x_4^2$ is given by

$$A = \begin{pmatrix} -1 & 2 & 1 & 0 \\ 2 & -5 & 0 & 0 \\ 1 & 0 & -6 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The leading principal minors are $D_1 = -1$, $D_2 = 5 - 4 = 1$, $D_3 = -6D_2 + 1(0 - (-5)) = -6 + 5 = -1$, and $D_4 = |A| = -1D_3 = 1$. Hence f is negative definite. The correct answer is alternative **D**.

Question 7.

We compute the first order derivatives to find stationary points, and find

$$4x^3 - 4y = 0, \quad -4x + 4y^3 = 0$$

This gives $y = x^3$ and $x = y^3 = (x^3)^3 = x^9$, or $x(1 - x^8) = 0$. There are three stationary points $(0,0), (1,1), (-1,-1)$ corresponding to the solutions $x = 0$, $x = 1$ and $x = -1$. We compute the Hessian matrix of f and find

$$H(f) = \begin{pmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{pmatrix}$$

At $(0,0)$ the matrix is indefinite since $D_2 = -16 < 0$. At the two other points, $D_1 = 12 > 0$ and $D_2 = 144 - 16 > 0$ so the matrix is positive definite. Hence $(1,1)$ and $(-1,-1)$ are local minima while $(0,0)$ is a saddle point. The correct answer is alternative **B**.

QUESTION 8.

The function $f(x, y, z) = x^4 + 4xy + y^4 + hz^4 + z^2$ has Hessian matrix

$$H(f) = \begin{pmatrix} 12x^2 & 4 & 0 \\ 4 & 12y^2 & 0 \\ 0 & 0 & 12hz^2 + 2 \end{pmatrix}$$

Hence $D_2 = 144x^2y^2 - 16$ take both positive and negative values, f is neither convex nor concave. The correct answer is alternative **D**.