BI NORWEGIAN BUSINESS SCHOOL

Solutions:	GRA 60352	Mathematics		
Examination date:	19.04.2013	09:00 – 10:00	Total no. of pages:	2
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Re-take exam			Responsible department: Economics	

Correct answers: C-B-B-A-C-D-C-C

Question 1.

We reduce the augmented matrix to echelon form after interchanging the rows:

$$\left(\begin{array}{cccc|c} 1 & -1 & -2 & 4 & 7 \\ 0 & 2 & -3 & 1 & 4 \\ -2 & 8 & -5 & -5 & -2 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & -1 & -2 & 4 & 7 \\ 0 & 2 & -3 & 1 & 4 \\ 0 & 6 & -9 & 3 & 12 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & -1 & -2 & 4 & 7 \\ 0 & 2 & -3 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

From the pivot positions, we see that the system has two degrees of freedom. The correct answer is alternative **C**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its rank. We see that it is the transpose of the coefficient matrix in Question 1, hence it has rank two. The determinant

$$\begin{vmatrix} 0 & -2 \\ 2 & 8 \end{vmatrix} = -4 \neq 0$$

shows that the vectors $\mathbf{v}_1, \mathbf{v}_2$ are linearly independent, and $\mathbf{v}_3$ is a linear combination of these vectors since the rank is two. Hence the correct answer is alternative **B**.

Question 3.

We reduce the matrix A to an echelon form:

$$\begin{pmatrix} 0 & 2 & -3 & h & 4 \\ -2 & 8 & -5 & -5 & -2 \\ 1 & -1 & -2 & 4 & 7 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & 0 & 0 & h-1 & 0 \\ 0 & 6 & -9 & 3 & 12 \\ 1 & -1 & -2 & 4 & 7 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & -1 & -2 & 4 & 7 \\ 0 & 6 & -9 & 3 & 12 \\ 0 & 0 & 0 & h-1 & 0 \end{pmatrix}$$

We see that the rank of A is three if $h \neq 1$, and two if $h = 1$. The correct answer is alternative **B**.

Question 4.

The characteristic equation of A is $\lambda^2 + \lambda - 12 = 0$, and therefore that it has eigenvalues $\lambda = 3$ and $\lambda = -4$. The correct answer is alternative **A**.

Question 5.

We see that $A\mathbf{u} = -4\mathbf{u}$ while $A\mathbf{v} \neq \lambda\mathbf{v}$ for any λ. The correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $Q(x_1, x_2) = hx_1^2 - 4x_1x_2 + 3x_2^2$ is

$$A = \begin{pmatrix} h & -2 \\ -2 & 3 \end{pmatrix}$$

The leading principal minors are $D_1 = h$ and $D_2 = 3h - 4$. If $h > 4/3$, then $D_1, D_2 > 0$ and Q is positive definite. If $h = 4/3$, then $D_1 = 4/3 > 0$ and $D_2 = 0$, with $\Delta_1 = 4/3, 3 \geq 0$, and Q is positive semidefinit. If $h < 4/3$, then $D_2 < 0$ and Q is indefinite. The correct answer is alternative **D**.

Question 7.

We compute the Hessian matrix of $f(x, y) = x^4 + x^2 - 2xy + hy^2$ and find

$$H(f) = \begin{pmatrix} 12x^2 + 2 & -2 \\ -2 & 2h \end{pmatrix}$$

The principal minors of order one are all equal to $12x^2 + 2, 2h$, and $D_2 = 24hx^2 + 4h - 4$. If $h > 1$, then $D_1, D_2 > 0$ and f is convex. If $h = 1$, then $D_2 \geq 0$ (and equal to zero at $x = 0$). We check all principal minors, and find that $\Delta_1 \geq 0$, so f is convex. If $h < 1$, then $D_2 < 0$ for some values of x and f is neither convex nor concave. The correct answer is alternative **C**.

Question 8.

The set S defined by $x^2 - y^2 + z^2 \leq 1$ and $x, y, z \geq 0$ is clearly closed, but it is not bounded since $(0, a, 0)$ lies in S for any value $a \geq 0$ since $-a^2 \leq 1$. The value $f(0, a, 0) = 2a \to \infty$ when $a \to \infty$, so f does not have a maximum on S. The correct answer is alternative **C**.