BI NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	07.02.2012	15:00 – 16:00	Total no. of pages:	2
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions are weighted equally	
Extraordinary re-sit exam			Responsible department: Economics	

Correct answers: A-A-D-B-C-B-C-B

Question 1.

We reduce the augmented matrix to echelon form:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & 1 & 1 & 3 \\ 0 & 1 & -1 & -1 & -4 \\ 0 & 1 & 1 & -1 & 2 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 5 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & 1 & 1 & 3 \\ 0 & 0 & 2 & 0 & 5 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right)$$

From the pivot positions, we see that the system is inconsistent. The correct answer is alternative **A**.

Question 2.

We form the matrix A with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and reduce A to an echelon form:

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & 32 \\ 7 & 3 & 16 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -7 & 23 \\ 0 & -11 & -5 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -7 & 23 \\ 0 & 0 & * \end{pmatrix}$$

where $* = -5 - 11 \cdot 23/7 \neq 0$. From the pivot positions, we see that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent. Hence the correct answer is alternative **A**.

Question 3.

We reduce the matrix A to an echelon form:

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 2 & 1 & -1 & 2 \\ 7 & 8 & -1 & h \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & -6 & 13 & h-7 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 7 & h-7 \end{pmatrix}$$

We see that the rank of A is three for all values of h, and the correct answer is alternative **D**.

Question 4.

The characteristic equation of A is $\lambda^2 - 12\lambda + 32 = 0$, and therefore the eigenvalues are $\lambda = 4$ and $\lambda = 8$. The correct answer is alternative **B**.

Question 5.

The eigenvalues of A are $\lambda = 1$ (with multiplicity two) and $\lambda = 2$, since we have

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & h & h^2 \\ 0 & 2-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^2(2-\lambda) = 0$$

We compute the eigenvectors of $\lambda = 1$, the eigenvalue of multiplicity 2, by reducing the matrix $A - I$ to an echelon form:

$$\begin{pmatrix} 0 & h & h^2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & 0 & h^2-4h \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & 1 & 4 \\ 0 & 0 & h^2-4h \\ 0 & 0 & 0 \end{pmatrix}$$

We see that there are two degrees of freedom if $h = 0, 4$, and one degree of freedom otherwise. Therefore, A is diagonalizable if and only if $h = 0, 4$, and the correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $Q(x_1, x_2) = -4x_1^2 + 24x_1x_2 - 36x_2^2$ is

$$A = \begin{pmatrix} -4 & 12 \\ 12 & -36 \end{pmatrix}$$

The principal minors are $\Delta_1 = -4, -36$ and $\Delta_2 = 0$. Therefore A is negative semidefinite but not negative definite, and the correct answer is alternative **B**.

Question 7.

We compute the Hessian matrix of $f(x, y, z) = -e^{x+y+z}$: First, we compute the first order partial derivatives

$$f'_x = f'_y = f'_z = -e^{x+y+z}$$

and then we compute the second order partial derivatives and form the Hessian matrix

$$H(f) = \begin{pmatrix} -e^{x+y+z} & -e^{x+y+z} & -e^{x+y+z} \\ -e^{x+y+z} & -e^{x+y+z} & -e^{x+y+z} \\ -e^{x+y+z} & -e^{x+y+z} & -e^{x+y+z} \end{pmatrix}$$

The principal minors are $\Delta_1 = -e^{x+y+z}, -e^{x+y+z}, -e^{x+y+z}$, $\Delta_2 = 0, 0, 0$ and $\Delta_3 = 0$. Since $-e^{x+y+z} < 0$, it follows that f is concave but not convex. The correct answer is alternative **C**.

Question 8.

The set $S = \{(x, y) : 1 \leq x^2 + y^2 \leq 4\}$ of $\mathbb{R}^2$ is shown as the shaded region in the figure. We see that it is closed, since the boundary points are the two circles given by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and these circles are included in the set. We also see that the set is bounded. It is not convex, since the set has a hole. The correct answer is alternative **B**.