Exam	Final exam in GRA 6035 Mathematics
Date	January 9th 2026 at 0900 - 1400

You must give reasons for your answers.

Question 1.

(a) **(3p)** Find the general solution of the differential equation $y' - 3y = 12$.

(b) **(3p)** Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 3 \\ 3 & 10 & 5 \end{pmatrix}$$

(c) **(3p)** Determine the minimum value of $f(x, y) = 5x^2 - 6xy + 2y^2 - 4x + 2y + 3$.

(d) **(3p)** Determine $||\mathbf{v} - \mathbf{w}||$ when $\mathbf{v}, \mathbf{w}$ are vectors with $||\mathbf{v}|| = 2$, $||\mathbf{w}|| = 3$ and $\mathbf{v} \cdot \mathbf{w} = 2$.

Question 2.

We consider the matrices A and B given by

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

(a) **(6p)** Find the rank of A and a base of the column space of A.

(b) **(6p)** Express one of the column vectors of A as a linear combination of the others.

(c) **(6p)** Find the eigenvalues of the matrix B. Is B diagonalizable?

Question 3.

We consider the quadratic form u given by

$$u(x, y, z) = 4x^2 + 4xy + 2xz + y^2 + 4yz + z^2$$

(a) **(6p)** Determine the definiteness of the quadratic form u.

(b) **(6p)** Find the stationary points of the function $f(u) = e^u + 4e^{-u}$ in one variable.

(c) **(6p)** Determine the range of the function $f(x, y, z) = f(u)$, where $u = u(x, y, z)$.

Question 4.

(a) **(6p)** Solve the difference equation $y_{t+2} - 5y_{t+1} + 6y_t = 2$.

(b) **(6p)** Find the general solution $\mathbf{y} = (y_1, y_2, y_3)$ of the linear system of differential equations:

$$\mathbf{y}' = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & -2 \\ 3 & 6 & -2 \end{pmatrix} \cdot \mathbf{y}$$

(c) **(6p)** Solve the initial value problem $t^2 y' = (1 - y)^2,\ y(1) = 0$.

Question 5.

We consider the following Lagrange problem with parameter a:

$$\max f(x, y, z, w) = x + 2y + 4z + w + 2xw \text{ when } 3x^2 + y^2 + 4z^2 + w^2 = a$$

(a) **(6p)** Write down the Lagrange conditions, and find all candidate points (if any) with $\lambda = 0$.

(b) **(6p)** Determine when there are candidate points with $\lambda = 1$ that satisfies the Lagrange conditions, and find all corresponding candidates points.

(c) **(6p)** Find the maximum value $f^*(a)$ for the values of a found in (b), and use this to estimate the maximum value $f^*(4)$.