

You must give reasons for your answers.

Question 1.

- (a) **(3p)** Find the general solution of the difference equation $y_{t+1} - 2y_t = 200$.
- (b) **(3p)** Determine the definiteness of the quadratic form $f(x, y, z) = x^2 - xz + yz$.
- (c) **(3p)** Find the stable equilibrium states of the differential equation $y' = y(1 - y^2)$.
- (d) **(3p)** Determine all values of the parameter s such that A is diagonalizable:

$$A = \begin{pmatrix} 1 & s & 1 \\ 0 & 2 & s \\ 0 & 0 & 2 \end{pmatrix}$$

Question 2.

We consider the matrix A and the quadratic form $f(\mathbf{x}) = \mathbf{x}^T A \mathbf{x}$ when

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

- (a) **(6p)** Compute the trace and the determinant of A .
- (b) **(6p)** For each of the numbers $\lambda = 1, 2, 4$, determine whether λ is an eigenvalue of A .
- (c) **(6p)** Determine the range of f .
- (d) **(6p)** Determine whether the set $D = \{(x, y, z) : f(x, y, z) \geq 8\}$ is compact.

Question 3.

Let $u(x, y, z) = xy - xz - yz$ and consider the function

$$f(x, y, z) = e^u + e^{-u} \text{ with } u = u(x, y, z)$$

- (a) **(6p)** Determine the range of u .
- (b) **(6p)** Find the maximum and minimum values of f , if they exist.

Question 4.

- (a) **(6p)** Find the general solution of the differential equation $y'' - 7y' - 8y = 1 - 6t - 8t^2$.
- (b) **(6p)** Find the solution of the differential equation $4t^2 y^3 y' = 1$ that satisfies $y(1) = 1$.
- (c) **(6p)** Find the general solution of the differential equation $2y - 3t^2 + 2(t + 1)y' = 0$.
- (d) **(6p)** Find the general solution of the system of difference equations $\mathbf{y}_{t+1} = A\mathbf{y}_t$ when

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Question 5.

Consider the Kuhn-Tucker problem

$$\min f(x, y, z) = x^2 - xy + xz \text{ when } y^2 - yz + z^2 \leq 12$$

- (a) **(6p)** Write down the Kuhn-Tucker conditions for this problem.
- (b) **(6p)** Find the minimum value in the Kuhn-Tucker problem, if it exists.