Exam	Final exam in GRA 6035 Mathematics
Date	November 28th 2023 at 0900 - 1400

You must give reasons for your answers.

Question 1.

(a) **(3p)** Find the general solution of the difference equation $y_{t+2} - y_{t+1} - 2y_t = 0$.

(b) **(3p)** Find the equilibrium state of the Markov chain with transition matrix

$$A = \begin{pmatrix} 0.94 & 0.14 \\ 0.06 & 0.86 \end{pmatrix}$$

(c) **(3p)** In how many ways is it possible to write $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 7 \\ 10 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ -4 \\ -3 \end{pmatrix}$$

(d) **(3p)** Find the minimum value of $f(x, y, z) = x^2 + 2y^2 + 5z^2 - 4xz + 2x - 6z + 5$.

Question 2.

We consider the matrix A given by

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & 5 \\ 1 & 18 & 0 \end{pmatrix}$$

(a) **(6p)** Compute the rank and the determinant of A.

(b) **(6p)** Find a base of the null space of A.

(c) **(6p)** Determine the characteristic equation and the eigenvalues of A.

(d) **(6p)** Find the eigenvalues of B and determine the dimension of $\text{Null}(B)$ when $B = A^2$.

Question 3.

Let $f(x, y, z, w) = 27 - x^2 - 2y^2 + 2xz - 2z^2 + 2yw - 6w^2$ and consider the Lagrange problem

$$\max f(x, y, z, w) \text{ when } xw + yz = 10$$

(a) **(6p)** Determine whether the function f is concave.

(b) **(6p)** Find the candidate points $(x, y, z, w; \lambda)$ in the Lagrange problem with $\lambda = -2$.

(c) **(6p)** Show that the Lagrange problem has a maximum, and find the maximum value.

(d) **(6p)** Determine whether the set $\{(x, y, z, w) : xw + yz = 10\}$ of admissible points is compact.

Question 4.

(a) **(6p)** Find the general solution of the differential equation $y' + 4ty = 8t$.

(b) **(6p)** Find the particular solution of the differential equation that satisfy the initial condition:

$$y^2 - 2t + 2yt \cdot y' = 0, \quad y(1) = 2$$

(c) **(6p)** Find the general solution of the system $\mathbf{y}' = A\mathbf{y} + \mathbf{b}$ of differential equations, where

$$A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

(d) **(6p)** Solve the differential equation $y' + ty^2 = t$.