EXAMINATION QUESTION PAPER - Written examination

GRA 60353
Mathematics

Department of Economics

Start date:	08.01.2020	Time 13.00
Finish date:	08.01.2020	Time 16.00

Weight:	80% of GRA 6035
Total no. of pages:	2 incl. front page
Answer sheets:	Squares
Examination support materials permitted:	BI-approved exam calculator. Simple calculator. Bilingual dictionary.

Exam	Final exam in GRA 6035 Mathematics
Date	January 8th, 2020 at 1300 - 1600

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the matrix A, and the column vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ of A, given by

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & -1 & 3 & 0 \\ 1 & 7 & -6 & 9 \\ 5 & 0 & 5 & 3 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ -1 \\ 7 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 3 \\ -6 \\ 5 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 3 \\ 0 \\ 9 \\ 3 \end{pmatrix}$$

(a) **(6p)** Compute the rank of A.
(b) **(6p)** Find a base for $\text{Null}(A)$.
(c) **(6p)** Find a base $\mathcal{B}$ for $\text{Col}(A)$, and express $5\mathbf{v}_4$ as a linear combination of the vectors in $\mathcal{B}$.

Question 2.
We consider the matrix A and the vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ given by

$$A = \begin{pmatrix} -7 & 6 & 2 \\ -6 & 5 & 2 \\ -6 & 6 & 1 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

(a) **(6p)** Compute $\det(A)$.
(b) **(6p)** Show that $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are eigenvectors of A, and find their eigenvalues.
(c) **(6p)** Determine whether A is diagonalizable.

Question 3.

(a) **(6p)** Solve the differential equation $y'' - 11y' + 18y = 9t^2 - 11t + 10$.
(b) **(6p)** Solve the differential equation $e^t\, y' = t\, y^2$.
(c) **(6p)** Solve the linear system of differential equations:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 4 & -2 & 4 \\ 2 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Question 4.
We consider the Lagrange problem given by

$$\min\ f(x,y,z,w) = -4x^2 - 10y^2 - 5z^2 - 5w^2 + 4xz + 4xw - 4yz + 4yw + 6zw \text{ subject to } x^2+y^2+z^2+w^2 = 6$$

You may use that the Hessian matrix of f has determinant $\det H(f) = 0$.

(a) **(6p)** Determine whether f is convex or concave.
(b) **(6p)** Find all solutions $(x, y, z, w; \lambda)$ of the Lagrange conditions with $\lambda = -12$.
(c) **(6p)** Show that any solution in (b) solves the minimum problem.
(d) **Extra credit (6p)** Solve $\max f(x, y, z, w)$ subject to $x^2 + y^2 + z^2 + w^2 = 6$.