Written examination:	GRA 60353 Mathematics	
Examination date:	11.06.2013 09:00 – 12:00	Total no. of pages: 2
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus	
Answer sheets:	Squares	
	Counts 80% of GRA 6035	The subquestions have equal weight
Re-sit exam		Responsible department: Economics

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} 2 & 4 & s \\ -4 & -6 & -3 \\ s & s & 1 \end{pmatrix}$$

(a) Compute the determinant and rank of A.
(b) Compute all eigenvalues of A when $s = 0$. Is A diagonalizable when $s = 0$?

Question 2.

We consider the function $f(x, y; a) = xy^2 + 5x^3y - a^2xy$ with parameter a defined for all points $(x, y) \in \mathbb{R}^2$. We assume that $a > 0$.

(a) Compute the partial derivatives and the Hessian matrix of f.
(b) Compute all stationary points of f. Show that there is exactly one stationary point $(x^*(a), y^*(a))$ that is a local maximum, and find it.
(c) Will the local maximum value $f^*(a) = f(x^*(a), y^*(a))$ increase or decrease when the value of the parameter a increases?

Question 3.

Solve the following differential equations:

(a) $y'' = -15, \quad y(0) = 695,\ y'(0) = 55.5$
(b) $y' = (1 - 3t^2)y^2, \quad y(0) = -1$
(c) $(2y - t)e^{y^2 - yt}y' - ye^{y^2 - yt} = 0, \quad y(0) = 1$

Question 4.

We consider the following optimization problem:

$$\min\; xy^2 + 5x^3y - xy \text{ subject to } \begin{cases} x + y \leq 5 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

(a) Sketch the set of admissible points, and show that the optimization problem has a solution that satisfies the Kuhn-Tucker conditions.
(b) Solve the optimization problem and compute the corresponding minimum value.