

Key Problems

Problem 1.

Solve the differential equations:

a) $y' = 3t^2 + 2$

b) $ty' = 1$

c) $t^2y' = 1$

d) $y' = t\sqrt{t^2 + 1}$

e) $y' = 4t \ln t$

f) $y' = \frac{2t+8}{4-t^2}$

Problem 2.

Solve the differential equations:

a) $y' = 5y$

b) $y' = y^2t$

c) $y' = 5y(1 - y/10)$

Problem 3.

Solve the differential equations:

a) $y' + 3y = 6$

b) $y' - 2ty = 4t$

c) $y' + 2y = e^t$

Problem 4.

Solve the exact differential equations:

a) $3t^2 - 2t + 2y \cdot y' = 0$

b) $2y - 3t^2 + 2(y+t)y' = 0$

c) $\frac{y(1 - 2 \ln t)}{t^3} + \frac{\ln t}{t^2} \cdot y' = 0$

Problem 5.

Find the equilibrium states and determine their stability. Sketch the solution curves $y = y(t)$.

a) $y' = 6 - 2y$

b) $y' = y^2 - 4$

c) $y' = 5y(1 - y/10)$

Exercise problems

Problems from the textbook: [E] 7.1 - 7.25

Answers to Key Problems

Problem 1.

a) $y = t^3 + 2t + C$

b) $y = \ln |t| + C$

c) $y = -1/t + C$

d) $y = \frac{1}{3}(t^2 + 1)\sqrt{t^2 + 1} + C$

e) $y = 2t^2 \ln t - t^2 + C$

f) $y = \ln |2 + t| - 3 \ln |2 - t| + C$

Problem 2.

a) $y = Ke^{5t}$

b) $y = -2/(t^2 + 2C)$

c) $y = 10 \cdot Ke^{5t}/(1 + Ke^{5t})$

Problem 3.

a) $y = 2 + Ce^{-3t}$

b) $y = -2 + Ce^{t^2}$

c) $y = \frac{1}{3}e^t + Ce^{-2t}$

Problem 4.

a) $y = \pm\sqrt{t^2 - t^3 + C}$

b) $y = -t \pm \sqrt{t^2 + t^3 + C}$

c) $y = \frac{Ct^2}{\ln t}$

Problem 5.a) $y_e = 3$ is globally asymptotically stableb) $y_e = -2$ is stable (but not globally asymptotically stable), $y_e = 2$ is unstablec) $y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)