Key Problems

Problem 1.
We consider the function $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2 + 10z$.

a) Find all stationary points of f with $x = 1$.

b) Show that f has a global maximum point, and find the maximal value of f.

c) Use the envelope theorem to estimate $\max f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2 + 11z$.

Problem 2.
We consider the function $f(x,y,z) = 4x^2 + 4xy - 4xz + 5y^2 + 8z^2 + 36x + 18y - 72z + 200$.

a) Find the minimum value of f, if it exists.

b) Show that $f(x,y,z;a,b,c) = 4x^2 + 4xy - 4xz + 5y^2 + 8z^2 + ax + by + cz + 200$ has a minimum for all values of the parameters a,b and c.

c) Use the envelope theorem to estimate the minimum value of $f(x,y,z;a,b,c)$ when
i) $(a,b,c) = (35,18,-72)$ *ii)* $(a,b,c) = (36,20,-72)$ *iii)* $(a,b,c) = (36,18,-70)$ *iv)* $(a,b,c) = (35,20,-70)$

Problem 3.
We consider the constrained optimization problem $\max f(x,y,z) = 2x^2 - 4y^2 - 2z^2$ when $x^4 + y^4 + z^4 \leq 16$.

a) Find the maximum point and maximum value of f.

b) Use the envelope theorem to estimate the new maximum value of f when we change
i) the constraint to $x^4 + y^4 + z^4 \leq 20$ ii) the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$

Problem 4.
We consider the Lagrange problem given by

$$\min\ f(x,y,z,w) = -4x^2 - 10y^2 - 5z^2 - 5w^2 + 4xz + 4xw - 4yz + 4yw + 6zw \text{ when } x^2 + y^2 + z^2 + w^2 = 6$$

a) Determine whether f is convex or concave.

b) Find all points (x,y,z,w) such that $(x,y,z,w;\lambda)$ satisfy the Lagrange conditions when $\lambda = -12$.

c) Solve the Lagrange problem $\min f(x,y,z,w)$ subject to $x^2 + y^2 + z^2 + w^2 = 6$.

Exercise Problems

Problems from the textbook:	[E] 5.13 - 5.14, 6.8 - 6.9
Exam problems	[Final 01/2018] Question 1,3,4

Answers to Key Problems

Problem 1.

a) $(x,y,z) = (1,0,4/3)$

b) $f_{\text{max}} = 71/3$

c) $f^*(11) \approx 25$

Problem 2.

a) $f_{\text{min}} = 11$

b) f convex with unique stationary point

c) *i)* ≈ 13 *ii)* ≈ 9 *iii)* ≈ 19 *iv)* ≈ 19

Problem 3.

a) $(x,y,z;\lambda) = (\pm 2,0,0;1/4)$ with $f(\pm 2,0,0) = 8$

b) *i)* $f_{\text{max}} \cong 9$ *ii)* $f_{\text{max}} \cong 4$

Problem 4.

a) f is concave

b) $(0,-2,-1,1;-12), (0,2,1,-1;-12)$

c) $f_{\text{min}} = -72$