

## Key Problems

### Problem 1.

Determine the definiteness of the symmetric matrix:

$$\text{a) } A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$$

### Problem 2.

Find the symmetric matrix of the quadratic form, and determine its definiteness:

$$\text{a) } f(x,y) = x^2 - 8xy + 3y^2$$

$$\text{b) } f(x,y,z) = 2x^2 - 2xz + 3y^2 + z^2$$

$$\text{c) } f(x,y,z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$$

$$\text{d) } f(x,y,z,w) = xw - yz$$

### Problem 3.

Determine the definiteness of the symmetric matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

### Problem 4.

Find an orthogonal matrix  $P$  such that  $P^T A P = D$  is a diagonal matrix, and write down the quadratic form  $\mathbf{x}^T A \mathbf{x}$  in the the coordinates  $\mathbf{u} = P^T \mathbf{x}$ :

$$\text{a) } A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

### Problem 5.

Check that the Markov chain with transition matrix  $A$  is regular, and find its equilibrium state  $\mathbf{v}$ :

$$\text{a) } A = \begin{pmatrix} 0.40 & 0.15 \\ 0.60 & 0.85 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 0.77 & 0.46 \\ 0.23 & 0.54 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

$$\text{f) } A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$$

## Exercise Problems

Problems from the textbook: [E] 4.8 - 4.17

Exam problems: Midterm exam 10/2018 Question 1-7

## Answers to Key Problems

### Problem 1.

- a) Positive definite
- b) Negative definite
- c) Positive definite
- d) Positive semi-definite
- e) Indefinite

### Problem 2.

- a) Indefinite
- b) Positive definite
- c) Positive semi-definite
- d) Indefinite

### Problem 3.

Positive semi-definite

### Problem 4.

$$a) P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$2u^2 + 4v^2$$

$$b) P = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$3u^2 + 4v^2 + 5w^2$$

$$c) P = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & -1 & \sqrt{2} \\ 0 & 2 & \sqrt{2} \\ \sqrt{3} & -1 & \sqrt{2} \end{pmatrix}$$

$$-u^2 - v^2 + 2w^2$$

### Problem 5.

$$a) \mathbf{v} = \begin{pmatrix} 1/5 \\ 4/5 \end{pmatrix}$$

$$b) \mathbf{v} = \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix}$$

$$c) \mathbf{v} = \begin{pmatrix} 3/17 \\ 14/17 \end{pmatrix}$$

$$d) \mathbf{v} = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$$

$$e) \mathbf{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$$

$$f) \mathbf{v} = \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix}$$