

Key Problems

Problem 1.

Find the equilibrium states and determine their stability. Sketch the solution curves $y = y(t)$.

a) $y' = 6 - 2y$

b) $y' = y^2 - 4$

c) $y' = 5y(1 - y/10)$

Problem 2.

Find the general solution of the second order differential equations:

a) $y'' + 6y' - 16y = 16t - 22$

b) $y'' + 6y' + 9y = 4e^{-t}$

c) $y'' - 3y' + 2y = 3e^{2t}$

d) $y'' - y = t^2$

Problem 3.

Solve the first order differential equations using the i) superposition principle, and ii) integrating factors.

a) $y' = y - t$

b) $y' + y = e^{-2t}$

c) $y' + 2y = 4t^2$

Exercise Problems

Textbook problems: [E] 7.24 - 7.34

Exam problems [Final 12/2015] Q2, [Final 11/2017] Q2, [Final 11/2018] Q2ac, Q5,
[Final 01/2021] Q3bc

Answers to Key Problems

Problem 1.

a) $y_e = 3$ is globally asymptotically stable

b) $y_e = -2$ is stable (but not globally asymptotically stable), $y_e = 2$ is unstable

c) $y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)

Problem 2.

a) $y = C_1 e^{-8t} + C_2 e^{2t} + 1 - t$

b) $y = C_1 e^{-3t} + C_2 t e^{-3t} + e^{-t}$

c) $y = C_1 e^{2t} + C_2 e^t + 3t e^{2t}$

d) $y = C_1 e^t + C_2 e^{-t} - t^2 - 2$

Problem 3.

a) $y = C e^t + t + 1$

b) $y = C e^{-t} - e^{-2t}$

c) $y = C e^{-2t} + 2t^2 - 2t + 1$