How to work with the problem sets

a) Work on Key Problems first. They test your understanding of important theory presented in the lecture, and the methods you need to master. Exercise problems are further problems from the textbook and exams, with full solutions available in the workbook.

b) In Exercise Sessions (Fri/Mon 16-18) you can work on problems and get help.

Key Problems

Problem 1.
Solve the linear system $A\mathbf{x} = \mathbf{b}$ using Gaussian elimination:

a) $A = \begin{pmatrix} 1 & 3 & 4 \\ 5 & 1 & 8 \\ 4 & 5 & 9 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ 15 \\ 23 \end{pmatrix}$

b) $A = \begin{pmatrix} 4 & 5 & 3 & 11 \\ 2 & 5 & 0 & 3 \\ 3 & 2 & 3 & 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 23 \\ 17 \\ 14 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 1 & 3 & 1 & 5 & 18 \\ 2 & 4 & 2 & 9 & 31 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 8 \\ 28 \\ 48 \end{pmatrix}$

d) $A = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 4 & 1 \\ 7 & 7 & 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 13 \\ 11 \\ 17 \end{pmatrix}$

Problem 2.
Find the rank of the following matrix:

$$A = \begin{pmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 1 \\ 3 & 0 & 2 & 3 & 1 \\ 3 & 6 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & -1 \end{pmatrix}$$

Problem 3.
Let V be the set of solutions of the linear system $A\mathbf{x} = \mathbf{b}$ from Problem 1(b). What is the dimension of V? Describe V geometrically.

Problem 4.
We consider the following linear system. Find all solutions that satisfies $x + w = y + z$:

$$\begin{aligned} x + y + 2z + 4w &= 6 \\ x + 2y + 4z - 2w &= 9 \\ x + 3y + 9z + 7w &= 24 \end{aligned}$$

Problem 5.
We consider a 4×5 homogeneous linear system $A\mathbf{x} = \mathbf{0}$, where $\operatorname{rk} A = 4$. How many solutions are there?

Exercise Problems

Problems from the textbook: [E] 1.1 - 1.17

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Answers to Key Problems

Problem 1.

a) $(1,2,1)$

b) $(6+11t,1-5t,-2-10t,t)$

c) $(2-s,2-t,s,4-3t,t)$

d) No solutions

Problem 2.
$\text{rk}(A) = 5$

Problem 3.
We have that $\dim V = 1$ since there is one degree of freedom. The solution set V is a straight line in four-dimensional space $\mathbb{R}^4$.

Problem 4.
$(x,y,z,w) = (1,-9/5,3,1/5)$

Problem 5.
Infinitely many solutions (one degree of freedom).

Answer to Exercise Problems

You find full solutions to all Exercise problems in the workbook [EP] Eriksen, *Graduate mathematics for Business, Economics and Finance - Problems and Solutions*.