	Plan	
1	Key Problems:	1.4, 2.1bdf, 2.3, 2.4cd, 2.5bc, 3.2bc, 3.3b, 3.4b, 4.1def, 4.2, 4.3, 4.4, 4.5
2	Exam Problems:	Midterm 01/2020 Q4,Q8 10/2022 Q6,Q8

(1) Key problems

$x + w = y + z \rightarrow x - y - z + w = 0$

1.4
$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 6 \\ 1 & 2 & 4 & -2 & 9 \\ 1 & 3 & 9 & 7 & 24 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right)$$

Gaussian elimination

$-14 \cdot (-4/3) = \frac{56}{3}$

$-10 \cdot (-4/3) = \frac{40}{3}$

2.1
$$\begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 3 & 3 & 3 & 4 & 2 \\ 4 & 4 & 4 & 5 & 3 \end{pmatrix} \begin{matrix} -3 \\ -4 \end{matrix} \rightarrow \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix} -4/3$$

$$\rightarrow \begin{array}{c} \left(\begin{array}{cccc|c} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 0 & 0 & -1/3 & 1/3 \end{array}\right) \\ \begin{array}{ccccc} \underline{v_1} & \underline{v_2} & & & v_5 \end{array} \end{array}$$

d)
$$\begin{array}{ccc} \underline{v_1} & \underline{v_2} & \underline{v_5} \\ \hline \end{array}$$
$$\begin{pmatrix} 1 & -1 & 4 \\ 0 & 6 & -10 \\ 0 & 0 & 1/3 \end{pmatrix}$$

$\Rightarrow \underline{v_1}, \underline{v_2}, v_5$ are lin. independent

b) $x_1 \underline{v_1} + x_2 \underline{v_2} + x_3 \underline{v_3} + x_4 \underline{v_4} = \underline{v_5}$

$-\frac{1}{3} x_4 = \frac{1}{3} \Rightarrow x_4 = \underline{-1}$

$\frac{6x_2}{6} = -10 + 12x_3 + 14(-1) = \frac{-24 + 12x_3}{6}$ $\quad x_2 = \underline{-4 + 2x_3}$

$x_1 = 4 + (-4 + 2x_3) - 5x_3 - 6(-1) = \underline{6 - 3x_3}$

$\underline{x_3 = 0}$: $6\underline{v_1} - 4\underline{v_2} - \underline{v_4} = \underline{v_5}$ $\quad (6, -4, 0, -1)$

f) $\underline{x_3 = 2}$: $2\underline{v_3} - \underline{v_4} = \underline{v_5}$ $\quad (0, 0, 2, -1)$

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2.3.

$$\begin{array}{cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \end{array}$$

$$A = \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 2 & 4 & -2 & -2 & -2 \\ 3 & 5 & -1 & -1 & -1 \end{pmatrix} \begin{matrix} -2 \\ -3 \end{matrix}$$

$$\to \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix} -4/3 \to \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 0 & 0 & -1/3 & 1/3 \end{pmatrix} \left|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right.$$

$\text{Col}(A) = \text{span}(\underline{v}_1, \dots, \underline{v}_5)$ $\quad$ $\dim \text{Col}(A) = \text{rk}\, A = \underline{3}$

Base: $\{\underline{v}_1, \underline{v}_2, \underline{v}_4\}$

$\text{Null}(A)$: $A\underline{x} = \underline{0}$ $\quad$ $\dim \text{Null}(A) = 5 - \text{rk}(A) = \underline{2}$

Solutions: $-\frac{1}{3}x_4 + \frac{1}{3}x_5 = 0 \Rightarrow x_4 = x_5$

$$\frac{6x_2}{6} = \frac{12x_3 + 14(x_5) + 10x_5 = 12x_3 + 24x_5}{6}$$

$$x_2 = 2x_3 + 4x_5$$

$$x_1 = (2x_3 + 4x_5) - 5x_3 - 6(x_5) - 4x_5$$
$$= -3x_3 - 6x_5$$

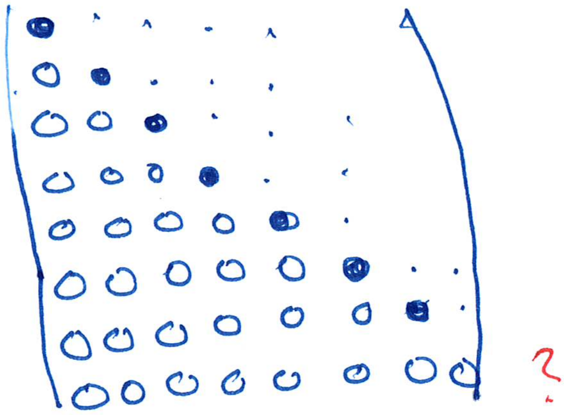
$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -3x_3 - 6x_5 \\ 2x_3 + 4x_5 \\ x_3 \\ x_5 \\ x_5 \end{pmatrix} = x_3 \cdot \underbrace{\begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{\underline{w}_1} + x_5 \underbrace{\begin{pmatrix} -6 \\ 4 \\ 0 \\ 1 \\ 1 \end{pmatrix}}_{\underline{w}_2}$$

$\text{Null}(A) = \text{span}(\underline{w}_1, \underline{w}_2)$

$\Rightarrow$ Base: $\{\underline{w}_1, \underline{w}_2\}$

2.4 cd

The echelon form $\underline{E}$ of A: $E =$



c) → ?

$? = 0$: one free var., inf. many solutions

$(A|\underline{b}) \to (E|*)$

$? \neq 0$: no solutions

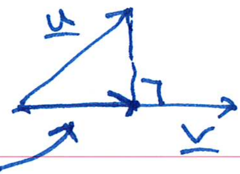
d) $\left(\begin{array}{ccccc|c} & & A & & & \underline{0} \\ \hline 1 & 1 & 1 & \cdots & 1 & 1 \end{array}\right) \to \left(\begin{array}{cccc|c} & & E & & \underline{0} \\ \hline 0 & 0 & \cdots 0 & ? & 1 \end{array}\right)$

$? = 0$: no sol's

$? \neq 0$: one solution

$\underline{u} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix} \quad \underline{v} = \begin{pmatrix} 2 \\ 4 \\ -1 \\ 2 \end{pmatrix} \quad \underline{w} = \begin{pmatrix} 1 \\ 2 \\ 4 \\ 2 \end{pmatrix}$

2.5 b) $\text{Proj}_{\underline{v}}(\underline{u}) = \dfrac{\underline{u}\cdot\underline{v}}{\underline{v}\cdot\underline{v}} \cdot \underline{v} = \dfrac{6}{25}\underline{v} = \dfrac{6}{25}\begin{pmatrix} 2 \\ 4 \\ -1 \\ 2 \end{pmatrix}$



c) $a : (\underline{v} - a\underline{w}) \perp \underline{w}$

$$(\underline{v} - a\underline{w})\cdot\underline{w} = 0 \Rightarrow \underline{v}\cdot\underline{w} - a\,\underline{w}\cdot\underline{w} = 0$$

$$\underline{v}\cdot\underline{w} = a(\underline{w}\cdot\underline{w})$$

$$10 = a \cdot 25$$

$$a = \frac{10}{25} = \underline{\underline{\frac{2}{5}}}$$

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3.2 b

$$\begin{vmatrix}0&1&3&0\\4&0&0&2\\2&0&0&4\\0&3&1&0\end{vmatrix} = -1\cdot\begin{vmatrix}4&0&2\\2&0&4\\0&1&0\end{vmatrix} + 3\cdot\begin{vmatrix}4&0&2\\2&0&4\\0&3&0\end{vmatrix}$$

$$= (-1)(-1)\cdot\begin{vmatrix}4&2\\2&4\end{vmatrix} + 3(-3)\begin{vmatrix}4&2\\2&4\end{vmatrix} = 12(1-9) = \underline{\underline{-96}}$$

3.3 b $M_{123,123} = 0 \qquad M_{123,124} \neq 0 \Rightarrow rk = \underline{\underline{3}}$

Base: $\{v_1, v_2, v_4\}$

3.4 b $M_{123,123} = \begin{vmatrix}1&3&2\\s&3&0\\4&6&2\end{vmatrix} = 2(6s-12)+2(3-3s)$

$= 12s - 24 + 6 - 6s = \underline{6s-18}$

$M_{123,123} = 0 \Leftrightarrow \left.\begin{matrix}6s-18=0\\ s=3\end{matrix}\right\} \Rightarrow \boxed{\begin{matrix}s\neq 3: & rk = \underline{3}\\ s=3: & rk = \underline{2}\end{matrix}}$

$\underline{s=3}$: $\begin{pmatrix}1&3&2&-1\\3&3&0&1\\4&6&2&0\end{pmatrix}$ $R1+R2 = R3$ $\Rightarrow rk = 2$

4.1 d) $A = \begin{pmatrix}3&0&1\\0&4&0\\1&0&3\end{pmatrix}$: $\begin{vmatrix}3-\lambda&0&1\\0&4-\lambda&0\\1&0&3-\lambda\end{vmatrix} = 0$ $\quad (4-\lambda)\cdot\begin{vmatrix}3-\lambda&1\\1&3-\lambda\end{vmatrix} = 0$

$\underline{\lambda_1 = 4}$, $\lambda^2 - 6\lambda + 8 = 0$

$\underline{\lambda_2 = 4}, \underline{\lambda_3 = 2}$

4.2 E_4: $\begin{pmatrix}-1&0&1\\0&0&0\\ \cancel{1}&\cancel{0}&\cancel{-1}\end{pmatrix}$ $x = z$, y, z free

$\underline{x} = \begin{pmatrix}z\\y\\z\end{pmatrix} = \underbrace{y\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}}_{\underline{v_1}} + \underbrace{z\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}}_{\underline{v_2}}$

Base: $\{\underline{v_1}, \underline{v_2}\}$

$D = \begin{pmatrix}4&0&0\\0&4&0\\0&0&2\end{pmatrix}$

$\underline{E_2}$: $\begin{pmatrix}1&0&1\\0&2&0\\ \cancel{1}&\cancel{0}&\cancel{1}\end{pmatrix}$ $x = -z$, $y = 0$, z free $\quad \underline{x} = \begin{pmatrix}-z\\0\\z\end{pmatrix} = \underbrace{z\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}}_{\underline{v_3}}$

Base: $\{\underline{v_3}\}$

$P = \begin{pmatrix}0&1&-1\\1&0&0\\0&1&1\end{pmatrix}$

Yes!

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e) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$: $\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\lambda = -1$ eigenvalue of mult. 2

$\Downarrow$

$\underline{\lambda_1 = \lambda_2 = -1,\ \lambda_3 = 2}$

$\underline{E_{-1}}$: $\begin{pmatrix} 1 & 1 & 1 \\ \cancel{1} & \cancel{1} & \cancel{1} \\ \cancel{1} & \cancel{1} & \cancel{1} \end{pmatrix}$ $x = -y - z$, y, z free

$\begin{pmatrix} -y-z \\ y \\ z \end{pmatrix} = y \cdot \underbrace{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}_{\underline{v_1}} + z \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{v_2}$

$\underline{\text{Base}}$

$\{\underline{v_1}, \underline{v_2}\}$

$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$P = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$\underline{\text{Yes}}$.

$\underline{E_2}$: $\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ \cancel{0} & \cancel{3} & \cancel{-3} \end{pmatrix}$

z free

$-3y + 3z = 0 \Rightarrow y = z$

$x = 2(z) - z = z$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ z \\ z \end{pmatrix} = z \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\underline{v_3}}$

$\underline{\text{Base}}$:

$\{\underline{v_3}\}$

f) $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$: $\begin{vmatrix} 0-\lambda & 1 & 1 \\ 0 & 0-\lambda & 1 \\ 0 & 0 & 0-\lambda \end{vmatrix} = 0$

A upper triangular $\Rightarrow$ eigenvalues = diagonal entries of A

$(0-\lambda)(0-\lambda)(0-\lambda) = 0$

$\underline{\lambda_1 = 0},\ \underline{\lambda_2 = 0},\ \underline{\lambda_3 = 0}$

$\underline{E_0}$: $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ x free

$\left.\begin{matrix} y + z = 0 \\ z = 0 \end{matrix}\right\}$ $\begin{matrix} y = 0 \\ z = 0 \end{matrix}$

$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = x \cdot \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\underline{v_1}}$

$\underline{\text{Base}}$:

$\{\underline{v_1}\}$

$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$P = \begin{pmatrix} 1 & ? & ? \\ 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \end{pmatrix}$ $\underline{\text{No}}$.

4.3 A symm $\Rightarrow$ A diag.

$$\begin{vmatrix} 1-\lambda & 0 & 0 & 4 \\ 0 & 2-\lambda & 3 & 0 \\ 0 & 3 & 2-\lambda & 0 \\ 4 & 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\cdot\begin{vmatrix} \boxed{\begin{matrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1-\lambda \end{vmatrix}$$

$$-4\cdot\begin{vmatrix} \begin{matrix} 0 \\ 0 \end{matrix} & \boxed{\begin{matrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{matrix}} \\ 4 & \begin{matrix} 0 & 0 \end{matrix} \end{vmatrix}$$

$$= \underline{(1-\lambda)\cdot(1-\lambda)}\cdot\begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} \underline{-4\cdot 4}\cdot\begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix}\cdot\left((1-\lambda)^2-16\right) = 0$$

$$(\lambda^2-4\lambda-5)(\lambda^2-2\lambda-15) = 0$$

$$\underline{\lambda=5,\ \lambda=-1\ ,\ \lambda=5,\ \lambda=-3}$$

4.4a $A = \begin{pmatrix} 0.4 & 0.15 \\ 0.6 & 0.85 \end{pmatrix}$: $\begin{vmatrix} 0.4-\lambda & 0.15 \\ 0.6 & 0.85-\lambda \end{vmatrix} = 0$

$$\lambda^2 - 1.25\lambda + 0.25 = 0$$

$0.4\cdot 0.85 - 0.6\cdot 0.15$

$= 0.34 - 0.09 = 0.25$

$$\underline{\lambda=1},\ \lambda=\underline{0.25}$$

$\underline{E_1}$: $\begin{pmatrix} -0.6 & 0.15 \\ \cancel{0.6} & \cancel{-0.15} \end{pmatrix} \to \begin{pmatrix} -6 & 1.5 \\ 0 & 0 \end{pmatrix}$ $-6x+1.5y=0$, y free $\quad \underline{v_1} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\underline{E_{0.25}}$: $\begin{pmatrix} 0.15 & 0.15 \\ \cancel{0.6} & \cancel{0.6} \end{pmatrix}$ $0.15x+0.15y=0$, y free $\quad \underline{v_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 0 \\ 0 & 0.25 \end{pmatrix}$ $\quad P = \begin{pmatrix} 1 & -1 \\ 4 & 1 \end{pmatrix} \Rightarrow D^N \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad P^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix}$

$$\lim_{N\to\infty} A^N = \lim_{N\to\infty} P D^N \cdot P^{-1} = \begin{pmatrix} 1 & -1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 4 & 0 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 1/5 & 1/5 \\ 4/5 & 4/5 \end{pmatrix}}}$$

4.5 a)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 0 \\ 3 & 5 & 1 \end{pmatrix} : \quad -\lambda^3 + c_1 \cdot \lambda^2 - c_2 \cdot \lambda + c_3 = 0$$

$c_1 = tr(A)$, $c_2 = M_{12,12} + M_{13,13} + M_{23,23}$, $c_3 = det(A)$

$$c_1 = tr(A) = 1+4+1 = \underline{6}$$

$$c_2 = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 0 \\ 5 & 1 \end{vmatrix}$$

$$= 2 - 2 + 4 = \underline{4}$$

$$c_3 = |A| = 1\cdot(10-12) + 1\cdot(4-2)$$

$$= -2 + 2 = \underline{0}$$

$$-\lambda^3 + 6\lambda^2 - 4\lambda = 0$$

$$-\lambda(\lambda^2 - 6\lambda + 4) = 0$$

$\underline{\lambda = 0}$ or $\lambda^2 - 6\lambda + 4 = 0$

$$\lambda = \frac{6 \pm \sqrt{36-16}}{2}$$

$$= 3 \pm \frac{\sqrt{20}}{2}$$

$$= 3 \pm \frac{\sqrt{4}\cdot\sqrt{5}}{2}$$

$$= \underline{3 \pm \sqrt{5}}$$

b)

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} :$$

$c_2 = 5 + 8 + 5$

$c_3 = 3 \cdot 5 - 2 \cdot 4 + 1 \cdot 1$

$$-\lambda^3 + 9\lambda^2 - 18\lambda + 8 = 0$$

$\lambda = 2$: $-8 + 36 - 36 + 8 = 0$

$\Rightarrow (\lambda - 2)$ is factor

If there is an int. solution, then it must divide 8, ie $\pm 1, \pm 2, \pm 4, \pm 8$

$$(-\lambda^3 + 9\lambda^2 - 18\lambda + 8) : (\lambda - 2) = -\lambda^2 + 7\lambda - 4$$

$$-\lambda^3 + 2\lambda^2$$

$$7\lambda^2$$

$$7\lambda^2 - 14\lambda$$

$$-4\lambda$$

$$-4\lambda + 8$$

$$0$$

$$\Rightarrow (\lambda - 2)(-\lambda^2 + 7\lambda - 4) = 0$$

$\underline{\lambda = 2}$ or $-\lambda^2 + 7\lambda - 4 = 0$

$$\lambda = \frac{-7 \pm \sqrt{49-16}}{-2}$$

$$= \underline{\frac{7}{2} \pm \frac{\sqrt{33}}{2}}$$

Quiz	Quiz 1 in GRA 6035 Mathematics
Date	September 16th, 2024

Question 1.
The homogeneous 4×5 linear system $A\mathbf{x} = \mathbf{0}$ has two degrees of freedom. What is the rank of A?

Question 2.
Give a precise definition of the null space of a matrix.

Question 3.
The point $(x, y, z, w) = (1, 0, 3, 2)$ is a solution of a 3×4 homogeneous linear system $A\mathbf{x} = \mathbf{0}$, where one of the minors of A is $M_{123,134} = 1$. Find all solutions of $A\mathbf{x} = \mathbf{0}$ with $xw - yz = 8$.

Question 4.
Determine the dimension of the eigenspace E_λ of A which contains the vector $\mathbf{v}$:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Q1: $\operatorname{rk} A = 3$

Q2: The nullspace of a matrix A is the set of all solution of $A\underline{x} = \underline{0}$.

Q3: One free var, homogeneous $\Big\} \Rightarrow (x,y,z,w) = t \cdot (1,0,3,2)$

$x = t,\ y = 0,\ z = 3t,\ w = 2t$

$xw - yz = 2t^2 = 8$

$t^2 = 4$

$t = \pm 2$

$(x,y,z,w) = (2,0,6,4),\ (-2,0,-6,-4)$

Q4: $A \cdot \underline{v} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = \lambda \cdot \underline{v} \Rightarrow \underline{\lambda = 2}$

$E_2: \begin{pmatrix} \boxed{\begin{matrix} -1 & 1 \\ 0 & -1 \end{matrix}} & \begin{matrix} 0 \\ 1 \end{matrix} \\ \begin{matrix} 1 & 0 \end{matrix} & -1 \end{pmatrix}$

$\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1 \neq 0$

$\operatorname{rk} = 2$, one free

$\dim E_2 = \underline{1}$