Key Problems

Problem 1.
We consider the constrained optimization problem max $f(x,y,z) = 2x^2 - 4y^2 - 2z^2$ when $x^4 + y^4 + z^4 \leq 16$.

a) Find the maximum point and maximum value of f.
b) Use the envelope theorem to estimate the new maximum value of f when we change
i) the constraint to $x^4 + y^4 + z^4 \leq 20$ ii) the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$

Problem 2.
Determine the range of the following quadratic functions:

a) $f(x,y,z) = x^2 + 4xz + y^2 + 5z^2 - 4y + 2z$
b) $f(x,y,z,w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 6z^2 + 3w^2 + 1$

Problem 3.
We consider the Lagrange problem given by

$$\min\ f(x,y,z,w) = -4x^2 - 10y^2 - 5z^2 - 5w^2 + 4xz + 4xw - 4yz + 4yw + 6zw \text{ when } x^2 + y^2 + z^2 + w^2 = 6$$

a) Determine whether f is convex or concave.
b) Find all points (x,y,z,w) such that $(x,y,z,w;\lambda)$ satisfy the Lagrange conditions when $\lambda = -12$.
c) Solve max $f(x,y,z,w)$ subject to $x^2 + y^2 + z^2 + w^2 = 6$.

Problem 4.
Let $g(x,y,z,w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 7z^2 + 4w^2$, and consider the Kuhn-Tucker problem given by

$$\max\ f(x,y,z) = x + y + z + w \text{ subject to } g(x,y,z,w) \leq 18$$

a) Determine the definiteness of the quadratic form g.
b) Write down the Kuhn-Tucker conditions of the problem in matrix form.
c) Write down the non-degenerate constraint qualification in this problem, and find all admissible points where this condition does not hold (if there are any).
d) Solve the Kuhn-Tucker problem.
e) Determine whether the set $D = \{(x,y,z,w) : g(x,y,z,w) \leq 18\}$ of admissible points is a compact set.

Exercise Problems

Exam problems [Final 01/2018] Question 1,3,4

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Answers to Key Problems

Problem 1.

a) $(x,y,z;\lambda) = (\pm 2,0,0;1/4)$ with $f(\pm 2,0,0) = 8$

b) *i)* $f_{\max} \cong 9$ *ii)* $f_{\max} \cong 4$

Problem 2.

a) $[-5,\infty)$

b) $[1,\infty)$

Problem 3.

a) f is concave

b) $(0,-2,-1,1;-12), (0,2,1,-1;-12)$

c) $f_{\max} = 0$

Problem 4.

a) positive definite

b) $\mathbf{e} - 2\lambda A\mathbf{x} = \mathbf{0},\ \mathbf{x}^T A\mathbf{x} \leq 18,\ \lambda \geq 0,\ \lambda(\mathbf{x}^T A\mathbf{x} - 18) = 0$

c) no admissible points where NDCQ does not hold

d) $f_{\max} = 6$

e) D is compact