

## Key Problems

### Problem 1.

We consider the quadratic form  $f(x,y,z,w) = x^2 + ay^2 - 2ayz + az^2 + w^2 - 2xw$  with parameter  $a$ . Determine the definiteness of  $f$  when

a)  $a > 0$

b)  $a = 0$

c)  $a < 0$

### Problem 2.

Solve the unconstrained optimization problem  $\max / \min f(\mathbf{x})$ :

a)  $f(x,y,z) = 5x^2 + 6xy + 2y^2 + 16xz + 10yz + 13z^2$

b)  $f(x,y,z,w) = 2xz - 2yw - x^2 - y^2 - z^2 - w^2$

c)  $f(x,y,z,w) = 2xy + 2xz + 2yw + 2zw$

d)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

### Problem 3.

Find all stationary points of  $f$ , classify them as local maximum/minimum points or saddle points, and determine whether  $f$  has global maximum/minimum values:

a)  $f(x,y,z) = xy + xz - yz$

b)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

c)  $f(x,y,z) = x^4 + y^4 + z^4 + z^2$

d)  $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

### Problem 4.

Determine whether  $f$  is a convex or concave function:

a)  $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

b)  $f(x,y,z) = e^{x-2y+z}$

c)  $f(x,y,z) = x^4 + y^4 + z^4 + z^2$

d)  $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

e)  $f(x,y,z) = \frac{xy + xz + yz}{xyz}$  defined for  $x,y,z > 0$

### Problem 5.

Determine the range  $V(f)$  of  $f$ :

a)  $f(x,y,z) = \ln(1 + 2x^2 + 2xy + 3y^2 - 2xz + z^2)$

b)  $f(x,y,z) = (x^2 + y^2 + z^2)e^{-x^2-y^2-z^2}$

## Exercise Problems

Problems from the textbook: [E] 5.1 - 5.12

Exam problems: [Midterm 10/2017] Question 1-8

## Answers to Key Problems

### Problem 1.

- $f$  is positive semidefinite when  $a > 0$
- $f$  is positive semidefinite when  $a = 0$
- $f$  is indefinite when  $a < 0$

### Problem 2.

- $f_{\min} = f(0,0,0) = 0$
- $f_{\max} = f(0,0,0,0) = 0$
- $f$  has no maximum or minimum
- $f_{\min} = f(0,0,0,0) = 0$

### Problem 3.

- Saddle point  $(0,0,0)$ , no global max/min value
- Local min  $(0,0,0,0)$ , global min value  $f_{\min} = 0$ , no global max value
- Local min  $(0,0,0)$ , global min value  $f_{\min} = 0$ , no global max value
- Local max  $(0,0,0)$ , global max value  $f_{\max} = 16$ , no global min value

### Problem 4.

- a) convex                      b) convex                      c) convex                      d) concave                      e) convex

### Problem 5.

- a)  $V(f) = [0, \infty)$               b)  $V(f) = [0, 1/e]$