Key Problems

Problem 1.
Compute A^{-1} and A^2:

a) $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ b) $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$

Problem 2.
Compute the determinant $|A|$:

a) $A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$ b) $A = \begin{pmatrix} 0 & 1 & 3 & 0 \\ 4 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 \\ 0 & 3 & 1 & 0 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & p & p^2 \\ 1 & q & q^2 \end{pmatrix}$

Problem 3.
Use minors to determine the rank of A, and find columns of A that form a base for the column space $\text{Col}(A)$. You may combine minors with other methods if it is difficult to answer the questions using only minors.

a) $A = \begin{pmatrix} 4 & 1 & 1 & 3 & 7 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 3 & 2 & 4 \\ 2 & -1 & 7 & 3 \\ 4 & 5 & 11 & 10 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & 4 & -3 & 1 \\ 2 & 7 & 1 & 2 \\ 1 & 3 & 4 & 1 \end{pmatrix}$

Problem 4.
Use minors to find the rank of these matrices:

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ t & t & t \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 3 & 2 & -1 \\ s & 3 & 0 & 1 \\ 4 & 6 & 2 & 0 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & a & b \\ a & b & c \end{pmatrix}$

Exercise Problems

Problems from the textbook:	[E] 3.1 - 3.15
Exam problems:	[Midterm 01/2020] Question 1, 2, 4, 8
	[Final 11/2019] Question 1

Answers to Key Problems

Problem 1.

a) $A^{-1} = \begin{pmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$, $A^2 = \begin{pmatrix} 1 & 4 & 4 \\ 2 & 5 & 4 \\ 2 & 6 & 5 \end{pmatrix}$

b) $A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$, $A^2 = \begin{pmatrix} 6 & 5 & 5 \\ 5 & 6 & 5 \\ 5 & 5 & 6 \end{pmatrix}$

c) A is not invertible, $A^2 = \begin{pmatrix} 1 & 4 & 4 \\ 2 & 7 & 6 \\ 3 & 11 & 10 \end{pmatrix}$

Problem 2.

a) $|A| = -23$

b) $|A| = -96$

c) $|A| = (p-1)(q-1)(q-p)$

Problem 3.

a) $\operatorname{rk} A = 3$, the column vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ (first three columns) is a base of $\operatorname{Col}(A)$

b) $\operatorname{rk} A = 3$, the column vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of $\operatorname{Col}(A)$

c) $\operatorname{rk} A = 2$, the column vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a base of $\operatorname{Col}(A)$

Problem 4.

a) $\operatorname{rk} A = 1$ for all t

b) $\operatorname{rk} A = \begin{cases} 2, & s = 3 \\ 3, & s \neq 3 \end{cases}$

c) $\operatorname{rk} A = \begin{cases} 1, & b = a^2 \text{ and } c = a^3 \\ 2, & \text{otherwise} \end{cases}$