Key Problems

Problem 1.
Solve the difference equations:

a) $y_{t+1} = 1.04y_t,\ y_0 = 100$
b) $y_{t+1} - 6y_t = 10t + 3$
c) $y_{t+2} - 3y_{t+1} + 2y_t = 0$
d) $y_{t+2} - 5y_{t+1} + 6y_t = 2t$
e) $y_{t+2} - 4y_{t+1} + 4y_t = 1$
f) $y_{t+2} + y_{t+1} - 2y_t = 6$

Problem 2.
Write the systems of difference equations on matrix form and solve them:

a) $\begin{pmatrix} y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_t \\ z_t \end{pmatrix}$
b) $\begin{pmatrix} y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 4 & 3 \end{pmatrix} \cdot \begin{pmatrix} y_t \\ z_t \end{pmatrix}$
c) $\begin{pmatrix} y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} y_t \\ z_t \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

Problem 3.
We consider a model for housing prices, where p_t is the price after t years. The model is given by the difference equation

$$p_{t+2} - 2p_{t+1} + p_t = -15, \quad p_0 = 695,\ p_1 = 743$$

a) Solve the difference equation.
b) We define $d_t = p_{t+1} - p_t$ to be the change in housing prices. Show that $d_{t+1} - d_t$ is constant, and use this to determine when housing prices will increase and when housing prices will decrease.

Problem 4.
Solve the systems of difference equations:

a) $\mathbf{y}_{t+1} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
b) $\mathbf{y}_{t+1} = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

Exercise Problems

Problems from the textbook	[E] 8.1 - 8.9, 9.8
Final exam problems	Final exam 11/2019 Q5, 01/2021 Q3a, 03/2021 Q3a

Answers to Key Problems

Problem 1.
a) $y_t = 100 \cdot 1.04^t$
b) $y_t = C \cdot 6^t - 2t - 1$
c) $y_t = C_1 + C_2 \cdot 2^t$
d) $y_t = C_1 \cdot 2^t + C_2 \cdot 3^t + t + 3/2$
e) $y_t = (C_1 + C_2 t) \cdot 2^t + 1$
f) $y_t = C_1 + C_2 \cdot (-2)^t + 2t$

Problem 2.

a) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} C_1\, 7^t - C_2\,(-3)^t \\ C_1\, 7^t + C_2\,(-3)^t \end{pmatrix}$ b) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} C_1\, 4^t - C_2\,(-1)^t \\ 4C_1\, 4^t + C_2\,(-1)^t \end{pmatrix}$ c) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} 4C_1\, 2^t - C_2\,(-3)^t + 3/4 \\ C_1\, 2^t + C_2\,(-3)^t - 3/4 \end{pmatrix}$

Problem 3.

a) $p_t = 695 + 55.5t - 7.5t^2$

b) $d_{t+1} - d_t = -15$, $d_t > 0$ for $t = 0, 1, 2, 3$ and that $d_t < 0$ for $t \geq 4$

Problem 4.

a) $\mathbf{y}_t = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot (-4)^t - \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot (-6)^t$ b) $\mathbf{y}_t = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \cdot 2^t + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot 3^t$