

Key Problems

Problem 1.

Find the equilibrium state \mathbf{v} of the Markov chains with transition matrix A . Is the Markov chain regular?

$$\text{a) } A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$$

Problem 2.

Determine the definiteness of the symmetric matrix:

$$\text{a) } A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$$

Problem 3.

Find the symmetric matrix of the quadratic form, and determine its definiteness:

$$\text{a) } f(x,y) = x^2 - 8xy + 3y^2$$

$$\text{b) } f(x,y,z) = 2x^2 - 2xz + 3y^2 + z^2$$

$$\text{c) } f(x,y,z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$$

$$\text{d) } f(x,y,z,w) = xw - yz$$

Problem 4.

Determine the definiteness of the symmetric matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

Problems from the Workbook

Workbook [W] 5.1 - 5.15 (full solutions in the workbook)

Exam problems Midterm exam 01/2018 Question 1-6, 8

Midterm exam 05/2018 Question 1-6, 8

Answers to Key Problems

Problem 1.

a) $\mathbf{v} = \begin{pmatrix} 3/17 \\ 14/17 \end{pmatrix}$ (regular)

b) $\mathbf{v} = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$ (regular)

c) $\mathbf{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$ (regular)

d) No equilibrium unless $\mathbf{v}_0 = \begin{pmatrix} a \\ b \\ a \end{pmatrix}$ (not regular)

e) $\mathbf{v} = \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix}$ (regular)

Problem 2.

a) Positive definite

b) Negative definite

c) Positive definite

d) Positive semi-definite

e) Indefinite

Problem 3.

a) Indefinite

b) Positive definite

c) Positive semi-definite

d) Indefinite

e) Indefinite

Problem 4.

Positive semi-definite