

Key Problems

Problem 1.

Check if the given sets are compact (closed and bounded), and if they are convex. It is useful to sketch the sets:

a) $D = \{(x,y) : x,y \geq 0 \text{ and } 2x + 3y \leq 6\}$

b) $D = \{(x,y) : 4x^2 + 9y^2 \leq 36\}$

c) $D = \{(x,y) : x,y \geq 1 \text{ and } 2x + 3y \geq 12\}$

d) $D = \{(x,y) : 4xyz \leq 1 \text{ and } x,y,z > 0\}$

Problem 2.

Determine whether the functions are convex or concave:

a) $f(x,y,z) = x - y + z$

b) $f(x,y,z) = 1 - e^{x-y+z}$

c) $f(x,y,z,w) = (x + y + z + w)^4$

d) $f(x,y) = |x - y|$

Problem 3.

Solve the Lagrange problems. You may assume that all admissible points satisfy the NDCQ:

a) $\max f(x,y,z) = x + 2y + 3z$ when $2x^2 + y^2 + 2z^2 = 9$

b) $\max / \min f(x,y,z) = x^4 + y^4 + z^4$ when $2x^2 + y^2 + 2z^2 = 9$

Problem 4.

Solve the Kuhn-Tucker problems. You may assume that all admissible points satisfy the NDCQ:

a) $\max f(x,y,z) = x - 2y + z$ when $x^2 + y^2 + z^2 \leq 3$

b) $\max f(x,y,z) = \ln(xyz)$ when $2x^2 + y^2 + 2z^2 \leq 6$

Problems from the Workbook

Workbook [W] 7.1 - 7.11 (full solutions in the workbook)
Exam problems Midterm exam 10/2018 Question 1-8

Answers to Key Problems

Problem 1.

- a) Compact and convex set
- b) Compact and convex set
- c) Convex, but not compact set (not bounded)
- d) Not convex and not compact set (not bounded)

Problem 2.

- a) Convex and concave
- b) Concave
- c) Convex
- d) Convex

Problem 3.

- a) $f_{\max} = 9$
- b) $f_{\max} = 81, f_{\min} = 9$

Problem 4.

- a) $f_{\max} = 3\sqrt{2}$
- b) $f_{\max} = \ln(2)/2$