Key Problems

Problem 1.
Determine the definiteness of the quadratic form f:

a) $f(x,y,z) = 5x^2 + 6xy + 2y^2 + 16xz + 10yz + 13z^2$

b) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + 2xz - 2yw$

c) $f(x,y,z,w) = 2xy + 2xz + 2yw + 2zw$

d) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

Problem 2.
Determine all values of a such that the symmetric matrix A is negative semidefinite:

$$A = \begin{pmatrix} a & 0 & 0 & -1 \\ 0 & a & -1 & 0 \\ 0 & -1 & a & 0 \\ -1 & 0 & 0 & a \end{pmatrix}$$

Problem 3.
Find all stationary points of f, classify them as local maximum/minimum points or saddle points, and determine whether f has global maximum/minimum values:

a) $f(x,y,z) = xy + xz - yz$

b) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

c) $f(x,y,z) = f(x,y,z) = x^4 + y^4 + z^4 + z^2$

d) $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

Problem 4.
Determine whether f is a convex or concave function:

a) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

b) $f(x,y,z) = e^{x-2y+z}$

c) $f(x,y,z) = x^4 + y^4 + z^4 + z^2$

d) $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

e) $f(x,y,z) = \dfrac{xy + xz + yz}{xyz}$ defined for $x,y,z > 0$

Problems from the Workbook

Workbook [W]	6.1 - 6.26 (full solutions in the workbook)	
Exam problems	Midterm exam 10/2017 Question 6-8,	Midterm exam 01/2018 Question 7
	Midterm exam 01/2019 Question 1-8,	Midterm exam 05/2018 Question 7

Answers to Key Problems

Problem 1.

a) Positive semi-definite b) Positive semi-definite c) Indefinite d) Positive definite

Problem 2.

It is negative semi-definite for $a \leq -1$.

Problem 3.

a) Saddle point (0,0,0), no global max/min value

b) Local min (0,0,0,0), global min value $f_{\min} = 0$, no global max value

c) Local min (0,0,0), global min value $f_{\min} = 0$, no global max value

d) Local max (0,0,0), global max value $f_{\max} = 16$, no global min value

Problem 4.

a) convex b) convex c) convex d) concave e) convex