Key Problems

In Problem 1-3, we consider the 3-vectors given by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}, \quad \mathbf{v}_5 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

Problem 1.
Determine if the vectors are linearly independent:

a) $\{\mathbf{v}_1, \mathbf{v}_2\}$ b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5\}$ d) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ e) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

Problem 2.
Compute the dimension of V, and find a base $\mathcal{B}$ of V:

a) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2)$ b) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ c) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5)$ d) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$

Problem 3.
Let $A = (\mathbf{v}_1|\mathbf{v}_2|\mathbf{v}_3|\mathbf{v}_4|\mathbf{v}_5)$ be the 3×5 matrix with $\mathbf{v}_1, \dots, \mathbf{v}_5$ as columns.

a) Compute $\dim \text{Null}(A)$, and find a base $\mathcal{B}$ for $\text{Null}(A)$.

b) Compute $\dim \text{Col}(A)$. What can you say about the linear subspace $\text{Col}(A)$ in $\mathbb{R}^3$ based on this?

Problem 4.
Find a parametric description of the line through the points (1,3,2,5) and (−2,4,5,1) in $\mathbb{R}^4$. Determine the intersection points (x,y,z,w) of this line and the hyperplane $w = 9$.

Problem 5.
Let A be a 5×7 matrix. Find $\dim \text{Col}(A) + \dim \text{Null}(A)$.

Problems from the Workbook

Workbook [W] 3.1 - 3.15 (full solutions in the workbook)

Answers to Key Problems

Problem 1.

a) Yes b) No c) Yes d) Yes e) No

Problem 2.

a) $\dim V = 2$, and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$

b) $\dim V = 2$, and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$

c) $\dim V = 3$, and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5\}$

d) $\dim V = 3$, and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$

Problem 3.

a) $\dim \text{Null}(A) = 2$ and $\mathcal{B} = \{\mathbf{w}_1, \mathbf{w}_2\}$ is a base for $\text{Null}(A)$ with

$$\mathbf{w}_1 = \begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{w}_2 = \begin{pmatrix} -6 \\ 4 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

b) $\dim \text{Col}(A) = 3$ and this means that $\text{Col}(A) = \mathbb{R}^3$.

Problem 4.

$$\text{Parametric description: } \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1-3t \\ 3+t \\ 2+3t \\ 5-4t \end{pmatrix}, \quad \text{Intersection point: } (x,y,z,w) = (4,2,-1,9)$$

Problem 5.

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