

Key Problems

Problem 1.

Write the systems of differential equations on matrix form and solve them:

$$\text{a) } y_1' = 2y_1 - 5y_2 \text{ and } y_2' = -5y_1 + 2y_2$$

$$\text{b) } y_1' = y_2 \text{ and } y_2' = 4y_1 + 3y_2$$

Problem 2.

Solve the systems of differential equations:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Problem 3.

Solve the systems of difference equations:

$$\mathbf{y}_{t+1} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}_t$$

Problems from the Workbook and Differential Equations

Workbook [W] 12.1 - 12.13 (full solutions in the workbook)

Differential equations [DE] 2.1 - 2.6 (full solutions on the web page)

Answers to Key Problems

Problem 1.

$$\text{a) } \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{-3t} - C_2 e^{7t} \\ C_1 e^{-3t} + C_2 e^{7t} \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{4t} - C_2 e^{-t} \\ 4C_1 e^{4t} + C_2 e^{-t} \end{pmatrix}$$

Problem 2.

$$\mathbf{y} = \begin{pmatrix} C_1 e^{-4t} & - & C_2 e^{-6t} \\ C_1 e^{-4t} & C_2 e^{-3t} & + \\ & & C_2 e^{-6t} \end{pmatrix}$$

Problem 3.

$$\mathbf{y}_t = \begin{pmatrix} C_1 (-4)^t & - & C_2 (-6)^t \\ C_1 (-4)^t & C_2 (-3)^t & + \\ & & C_2 (-6)^t \end{pmatrix}$$