GRA 6035 Mathematics

Problems for Lecture 12

Key problems

Problem 1.
Write the systems of differential equations on matrix form and solve them:
a) $y_1' = 2y_1 - 5y_2$ and $y_2' = -5y_1 + 2y_2$ *b)* $y_1' = y_2$ and $y_2' = 4y_1 + 3y_2$

Problem 2.
Solve the systems of differential equations:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Problem 3.
Find all equilibrium states in Problem 2. Are there globally asymptotically stable equilibrium states?

Problems from Differential Equations

Exercise problems 2.1 - 2.6 (full solutions on the web page)

Exam problems

Mock exam 11/2017 1 - 3 (full solutions on the web page)

Problems from the Digital Workbook

Exercise problems 12.1 - 12.13 (full solutions in the workbook)

Answers to key problems

Problem 1.

$$a)\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1\, e^{-3t} - C_2\, e^{7t} \\ C_1\, e^{-3t} + C_2\, e^{7t} \end{pmatrix} \quad b)\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1\, e^{4t} - C_2\, e^{-t} \\ 4C_1\, e^{4t} + C_2\, e^{-t} \end{pmatrix}$$

Problem 2.

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} C_1\, e^{-4t} & & - C_2\, e^{-6t} \\ & C_2\, e^{-3t} & \\ C_1\, e^{-4t} & & + C_2\, e^{-6t} \end{pmatrix}$$

Problem 3.
There is one equilibrium state $\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, and it is globally asymptotically stable since all eigenvalues are negative.