is $(4-2i)x = 4y$, or $x = 4y/(4-2i) = 2y/(2-i) = (4+2i)y/5$ with y free. When $a = 2-2i$, the first equation is $(4+2i)x = 4y$, or $x = 4y/(4+2i) = (4-2i)y/5$ with y free.

1.B Integration

Solution 1.B.1. We solve the indefinite integrals using basic integration rules:

a) $\int (3t^2 - 12t)\,\mathrm{d}t = t^3 - 6t^2 + C$

b) $\int (2\mathrm{e}^t - t)\,\mathrm{d}t = 2\mathrm{e}^t - t^2/2 + C$

c) $\int t\sqrt{t}\,\mathrm{d}t = \int t^{3/2}\,\mathrm{d}t = 2t^{5/2}/5 + C = 2t^2\sqrt{t}/5 + C$

d) $\int \frac{1}{t^3}\,\mathrm{d}t = \int t^{-3}\,\mathrm{d}t = t^{-2}/(-2) + C = -\frac{1}{2t^2} + C$

e) $\int (t-1)^2\,\mathrm{d}t = \int u^2\,\mathrm{d}u = u^3/3 + C = (t-1)^3/3 + C$

In the last integral, we have used the substitution $u = t - 1$, which gives $\mathrm{d}u = \mathrm{d}t$. We could also have expanded the parenthesis and integrated term by term:

$$\int (t-1)^2\,\mathrm{d}t = \int t^2 - 2t + 1\,\mathrm{d}t = \frac{1}{3}t^3 - t^2 + t + C$$

Solution 1.B.2. We have that

$$\int \frac{t^3 - t^2 + 1}{t}\,\mathrm{d}t = \int t^2 - t + \frac{1}{t}\,\mathrm{d}t = t^3/3 - t^2/2 + \ln|t| + C$$

Solution 1.B.3. We use integration by parts to compute the indefinite integrals:

a) We let $u' = t$ and $v = \ln t$, which gives $u = t^2/2$ and $v' = 1/t$:

$$\int t\ln(t)\,\mathrm{d}t = \frac{1}{2}t^2\ln t - \int \frac{1}{2}t^2\frac{1}{t}\,\mathrm{d}t = \frac{1}{2}t^2\ln t - \frac{1}{2}\int t\,\mathrm{d}t = \frac{1}{2}t^2\ln t - \frac{1}{4}t^2 + C$$

b) We let $u' = \mathrm{e}^t$ and $v = t$, which gives $u = \mathrm{e}^t$ and $v' = 1$:

$$\int t\,\mathrm{e}^t\,\mathrm{d}t = t\mathrm{e}^t - \int 1\cdot\mathrm{e}^t\,\mathrm{d}t = t\mathrm{e}^t - \int \mathrm{e}^t\,\mathrm{d}t = t\mathrm{e}^t - \mathrm{e}^t + C$$

c) We let $u' = \mathrm{e}^t$ and $v = t^2$, which gives $u = \mathrm{e}^t$ and $v' = 2t$:

$$\int t^2\mathrm{e}^t\,\mathrm{d}t = t^2\mathrm{e}^t - \int 2t\cdot\mathrm{e}^t\,\mathrm{d}t = t^2\mathrm{e}^t - 2\int t\mathrm{e}^t\,\mathrm{d}t$$

Using the result from b) we get

$$\int t^2 e^t \, dt = t^2 e^t - 2(te^t - e^t) + C = t^2 e^t - 2te^t + 2e^t + C$$

d) We write $\ln(t)/t^2 = t^{-2}\ln(t)$, and let $u' = t^{-2}$ and $v = \ln t$, which gives $u = -t^{-1}$ and $v' = 1/t$:

$$\int \frac{\ln(t)}{t^2} \, dt = -\frac{1}{t}\ln t - \int -\frac{1}{t} \cdot \frac{1}{t} \, dt = -\frac{1}{t}\ln t + \int t^{-2} \, dt = -\frac{1}{t}\ln t - \frac{1}{t} + C$$

e) We write $\sqrt{t}\ln(t) = t^{1/2}\ln(t)$, and let $u' = t^{1/2}$ and $v = \ln t$, which gives that $u = 2t^{3/2}/3$ and $v' = 1/t$:

$$\int \sqrt{t}\ln(t) \, dt = \frac{2}{3}t^{3/2}\ln t - \int \frac{2}{3}t^{3/2} \cdot \frac{1}{t} \, dt = \frac{2}{3}t\sqrt{t}\ln t - \frac{2}{3}\int t^{1/2} \, dt$$

We use that integral $\int \sqrt{t} \, dt$ again, and find that

$$\int \sqrt{t}\ln(t) \, dt = \frac{2}{3}t\sqrt{t}\ln t - \frac{4}{9}t\sqrt{t} + C$$

Solution 1.B.4. We use the substitution $u = \ln t$, which gives $du = (1/t)\,dt$, or that $dt = t\,du$:

$$\int \frac{\ln(t)}{t} \, dt = \int \frac{u}{t} t \, du = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln t)^2 + C$$

Solution 1.B.5. We use the substitution $u = 1 - t$, which gives $du = -dt$:

$$\int e^{1-t} \, dt = \int e^u(-du) = -e^u + C = -e^{1-t} + C$$

Solution 1.B.6. We use the substitution $u = at + b$, which gives $du = a\,dt$:

$$\int \frac{1}{at+b} \, dt = \int \frac{1}{u}\frac{du}{a} = \frac{1}{a}\int \frac{1}{u} \, du = \frac{1}{a}\ln|u| + C = \frac{1}{a}\ln|at+b| + C$$

Solution 1.B.7. We use substitution to solve the integrals:

a) With $u = t^2 + 1$, and $du = 2t\,dt$, we get

$$\int 3t\sqrt{t^2+1} \, dt = \int 3t\sqrt{u}\frac{du}{2t} = \frac{3}{2}\int u^{1/2} \, du = u^{3/2} + C = (t^2+1)\sqrt{t^2+1} + C$$

b) With $u = t^2 - 1$, and $du = 2t\,dt$, we get

$$\int \frac{t}{t^2-1} \, dt = \int \frac{t}{u}\frac{du}{2t} = \frac{1}{2}\int \frac{1}{u} \, du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|t^2-1| + C$$

c) With $u = t^2 - 1$, and $du = 2t\,dt$, we get

$$\int 5t(t^2-1)^3\,\mathrm{d}t = \int 5tu^3\,\frac{\mathrm{d}u}{2t} = \frac{5}{2}\int u^3\,\mathrm{d}u = \frac{5}{2}\frac{1}{4}u^4 + C = \frac{5}{8}(t^2-1)^4 + C$$

d) With $u = t^2+3t+2$, and $\mathrm{d}u = (2t+3)\,\mathrm{d}t$, we get

$$\int \frac{2t+3}{t^2+3t+2}\,\mathrm{d}t = \int \frac{2t+3}{u}\frac{\mathrm{d}u}{2t+3} = \int \frac{1}{u}\,\mathrm{d}u = \ln|u| + C = \ln|t^2+3t+2| + C$$

Solution 1.B.8. We use substitution to solve the integrals:

a) With $u = t^2$, and $\mathrm{d}u = 2t\,\mathrm{d}t$, we get

$$\int t\,\mathrm{e}^{t^2}\,\mathrm{d}t = \int t\,\mathrm{e}^u\,\frac{\mathrm{d}u}{2t} = \frac{1}{2}\int \mathrm{e}^u\,\mathrm{d}u = \frac{1}{2}\mathrm{e}^{t^2} + C$$

b) With $u = t^2$, and $\mathrm{d}u = 2t\,\mathrm{d}t$, we get

$$\int t^3\mathrm{e}^{t^2}\,\mathrm{d}t = \int t^3\mathrm{e}^u\,\frac{\mathrm{d}u}{2t} = \frac{1}{2}\int t^2\mathrm{e}^u\,\mathrm{d}u = \frac{1}{2}\int u\mathrm{e}^u\,\mathrm{d}u$$

Using that the integral $\int t\mathrm{e}^t\,\mathrm{d}t = t\mathrm{e}^t - \mathrm{e}^t + C$, we get

$$\int t^3\mathrm{e}^{t^2}\,\mathrm{d}t = \frac{1}{2}(u\mathrm{e}^u - \mathrm{e}^u) + C = \frac{1}{2}\mathrm{e}^{t^2}(t^2-1) + C$$

c) With $u = \sqrt{t}$, and $\mathrm{d}u = 1/(2\sqrt{t})\cdot\mathrm{d}t$, we get

$$\int \mathrm{e}^{\sqrt{t}}\,\mathrm{d}t = \int \mathrm{e}^u 2\sqrt{t}\,\mathrm{d}u = \int 2u\mathrm{e}^u\,\mathrm{d}u = 2(u\mathrm{e}^u - \mathrm{e}^u) + C = 2\sqrt{t}\mathrm{e}^{\sqrt{t}} - 2\mathrm{e}^{\sqrt{t}} + C$$

d) With $u = \sqrt{t}$, and $\mathrm{d}u = 1/(2\sqrt{t})\cdot\mathrm{d}t$, we get

$$\begin{aligned}\int \sqrt{t}\,\mathrm{e}^{\sqrt{t}}\,\mathrm{d}t &= \int u\mathrm{e}^u\cdot 2u\,\mathrm{d}u = 2\int u^2\mathrm{e}^u\,\mathrm{d}u = 2(u^2\mathrm{e}^u - 2u\mathrm{e}^u + 2\mathrm{e}^u) + C\\ &= 2t\mathrm{e}^{\sqrt{t}} - 4\sqrt{t}\mathrm{e}^{\sqrt{t}} + 4\mathrm{e}^{\sqrt{t}} + C\end{aligned}$$

e) With $u = \mathrm{e}^t$, and $\mathrm{d}u = \mathrm{e}^t\cdot\mathrm{d}t$, we get

$$\int \frac{2\mathrm{e}^t}{u+1/u}\frac{\mathrm{d}u}{\mathrm{e}^t} = \int \frac{2}{u+1/u}\,\mathrm{d}u = \int \frac{2u}{u^2+1}\,\mathrm{d}u$$

Finally, we use the substitution $v = u^2+1$, with $\mathrm{d}v = 2u\,\mathrm{d}u$, and this gives

$$\int \frac{2u}{u^2+1}\,\mathrm{d}u = \int \frac{2u}{v}\frac{\mathrm{d}v}{2u} = \int \frac{1}{v}\,\mathrm{d}v = \ln|v| + C = \ln|u^2+1| + C = \ln(\mathrm{e}^{2t}+1) + C$$

Solution 1.B.9. Using polynomial division, we rewrite the rational expression as

$$\frac{t^2-3t+7}{t-4} = t+1+\frac{11}{t-4}$$

The integral is given by

$$\int \frac{t^2-3t+7}{t-4}\,\mathrm{d}t = \int t+1+\frac{11}{t-4}\,\mathrm{d}t = \frac{1}{2}t^2+t+11\ln|t-4|+C$$

since the substitution $u=t-4$ gives $\mathrm{d}u=\mathrm{d}t$.

Solution 1.B.10. We use polynomial division, substitution, and partial fractions to compute the integrals:

a) Using polynomial division, we rewrite the rational expression as

$$\frac{t^2-3}{t+4} = t-4+\frac{13}{t+4}$$

This gives the integral

$$\int \frac{t^2-3}{t+4}\,\mathrm{d}t = \int t-4+\frac{13}{t+4}\,\mathrm{d}t = \frac{1}{2}t^2-4t+13\ln|t+4|+C$$

b) We use the substitution $u=t^2+2t+4$, with $\mathrm{d}u=(2t+2)\,\mathrm{d}t$:

$$\int \frac{t+1}{t^2+2t+4}\,\mathrm{d}t = \int \frac{t+1}{u}\,\frac{\mathrm{d}u}{2t+2} = \int \frac{1}{2u}\,\mathrm{d}u$$
$$= \frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|t^2+2t+4|+C$$

c) We use the substitution $u=t^2-4$, with $\mathrm{d}u=2t\,\mathrm{d}t$:

$$\int \frac{t}{t^2-4}\,\mathrm{d}t = \int \frac{t}{u}\,\frac{\mathrm{d}u}{2t} = \frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|t^2-4|+C$$

d) We use the method of partial fractions to rewrite the rational expression as

$$\frac{3}{t(3-t)} = \frac{A}{t}+\frac{B}{3-t}$$

This gives $3=A(3-t)+Bt$ after multiplying with the common denominator, or $3=3A+(B-A)t$. We see that $B-A=0$ and that $3A=3$, giving $A=B=1$. The integral is therefore given by

$$\int \frac{3}{t(3-t)}\,\mathrm{d}t = \int \frac{1}{t}+\frac{1}{3-t}\,\mathrm{d}t = \ln|t|-\ln|3-t|+C = \ln\left|\frac{t}{3-t}\right|+C$$

Notice the sign, which is result of the substitution $u=3-t$ with $\mathrm{d}u=-1\cdot\mathrm{d}t$.