Problems

Problem 5.1 Find the stationary points and classify their type:

a) $f(x, y) = x^4 + x^2 - 6xy + 3y^2$

b) $f(x, y) = x^2 - 6xy + 2y^2 + 10x + 2y$

c) $f(x, y) = xy^2 + x^3y - xy$

d) $f(x, y) = 3x^4 + 3x^2y - y^3$

e) $f(x, y) = e^{xy}$

f) $f(x, y) = \ln(x^2 + y^2 + 1)$

g) $f(x, y, z) = x^2 + 6xy + y^2 - 3yz + 4z^2 - 10x - 5y - 21z$

Problem 5.2 Find the stationary points and classify their type:

a) $f(x, y) = e^{xy}$ b) $f(x, y) = \ln(x^2 + y^2 + 1)$

Problem 5.3 Determine whether the function is convex or concave:

a) $f(x, y) = x^4 + x^2 - 6xy + 3y^2$

b) $f(x, y) = x^2 - 6xy + 2y^2 + 10x + 2y$

c) $f(x, y) = xy^2 + x^3y - xy$

d) $f(x, y) = 3x^4 + 3x^2y - y^3$

e) $f(x, y) = e^{xy}$

f) $f(x, y) = \ln(x^2 + y^2 + 1)$

g) $f(x, y, z) = x^2 + 6xy + y^2 - 3yz + 4z^2 - 10x - 5y - 21z$

Problem 5.4 Consider the subset $D = \{(x, y) : x \geq 0,\ y \geq 0,\ xy \leq 1\}$ of $\mathbb{R}^2$.

a) Sketch the set D

b) Describe the boundary points of D.

c) Determine if D is open or closed.

d) Determine if D is a convex set.

Problem 5.5 Let f be the function given by

$$f(x, y) = -6x^2 + (2a + 4)xy - y^2 + 4ay$$

where x, y are variables and a is a parameter. Determine the values of a such that f is a concave function.

Problems

Problem 6.1 Sketch each set and determine if it is open, closed, bounded or convex:

a) $\{(x, y) : x^2 + y^2 < 1\}$

b) $\{(x, y) : x^2 + y^2 \geq 2\}$

c) $\{(x, y) : xy \leq 1\}$

d) $\{(x, y, z) : x \geq 0, y \geq 0, z \geq 0\}$

e) $\{(x, y) : x \geq 0, y \geq 0, xy \geq 1\}$

f) $\{(x, y) : \ln(x) + \ln(y) \leq 5\}$

Problem 6.2 Determine whether the subset $D = \{(x, y, z, w) : xw - yz \leq -2\} \subseteq \mathbb{R}^4$ is compact.

Problem 6.3 Solve the following optimization problems:

a) $\max f(x, y) = xy$ subject to $2x + 3y = 12$

b) $\max f(x, y) = x^2y$ subject to $2x^2 + 5y^2 = 15$

c) $\max f(x, y) = xy$ subject to $x^2 + y^2 \leq 1$

d) $\min f(x, y, z) = x^2 + y^2 + z^2$ subject to $2x^2 + 6y^2 + 3z^2 \geq 36$

Problem 6.4 Solve the Lagrange problem

$$\max f(x, y, z) = xyz \quad \text{subject to} \quad \begin{cases} x^2 + y^2 = 1 \\ x + z = 1 \end{cases}$$

Problem 6.5 Solve the Kuhn-Tucker problem

$$\max f(x, y, z) = xyz \quad \text{subject to} \quad \begin{cases} x + y + z \leq 1 \\ x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$$

Problem 6.6 We consider the Kuhn-Tucker problem $\max f(x, y, z) = x^2yz$ subject to $x^2 + 2y^2 - 2z^2 \leq 32$.

a) Solve the Kuhn-Tucker conditions.

b) Does the maximum problem have a solution?

Problem 6.7 We consider the following optimization problem:

$$\max \ln(x^2y) - x - y \quad \text{subject to} \quad \begin{cases} x + y \geq 4 \\ x \geq 1 \\ y \geq 1 \end{cases}$$

Sketch the set of admissible points, and solve the optimization problem.