Problems

Problem 2.1 Show the vector $\mathbf{u} = (1, 3)$, $\mathbf{v} = (4, -1)$, and $\mathbf{w} = (-2, 2)$ in a figure, and use the geometric construction to find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{w}$, and $-2\mathbf{w}$. Then compute these vectors using coordinates.

Problem 2.2 Let $\mathbf{u} = (1, 3)$, $\mathbf{v} = (4, -1)$, and $\mathbf{w} = (-2, 2)$. Compute the following:

a) $\|\mathbf{u}\|$ b) $\|\mathbf{v}\|$ c) $\|\mathbf{w}\|$

d) $\mathbf{u} \cdot \mathbf{v}$ e) $\mathbf{u} \cdot \mathbf{w}$ f) $\mathbf{v} \cdot \mathbf{w}$

Problem 2.3 Let $\mathbf{u} = (1, 3)$, $\mathbf{v} = (4, -1)$, and $\mathbf{w} = (-2, 2)$. Compute the following:

a) $\mathrm{Proj}_{\mathbf{u}}(\mathbf{v})$ b) $\mathrm{Proj}_{\mathbf{u}}(\mathbf{w})$ c) $\mathrm{Proj}_{\mathbf{v}}(\mathbf{w})$ d) $\mathrm{Proj}_{\mathbf{v}}(\mathbf{u})$

Problem 2.4 Express the vector $\mathbf{w} = (8, 9)$ as a linear combination of $\mathbf{v}_1 = (2, 5)$ and $\mathbf{v}_2 = (-1, 3)$, if possible.

Problem 2.5 Find a parametric description of the following lines and planes:

a) The line through $(4, 1)$ and $(1, -3)$ in $\mathbb{R}^2$

b) The line in $\mathbb{R}^2$ with equation $2x + 3y = 1$

c) The line through $(1, 1, -1)$ and $(2, 0, 3)$ in $\mathbb{R}^3$

d) The plane through $(1, 1, -1)$, $(0, 3, 1)$ and $(2, 0, 3)$ in $\mathbb{R}^3$

e) The plane in $\mathbb{R}^3$ with equation $x + 2y + 3z = 6$

Problem 2.6 What is the distance from the origin to a point in $\mathbb{R}^3$ satisfying the equation $x^2 + y^2 + z^2 = 16$?

Problem 2.7 Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors of length $\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{w}\| = 2$ with

$$\mathbf{u} \cdot \mathbf{v} = 1, \quad \mathbf{u} \cdot \mathbf{w} = -2, \quad \mathbf{v} \cdot \mathbf{w} = 3$$

What is the length of the vector $\mathbf{u} - \mathbf{v} + 2\mathbf{w}$?

Problem 2.8 Determine whether the following pairs of vectors are linearly independent:

a) $\mathbf{v} = (-1, 2)$, $\mathbf{w} = (3, -6)$ b) $\mathbf{v} = (2, -1)$, $\mathbf{w} = (3, 4)$

Problems

Problem 3.1 Let A, B, and C be the 3×3 matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

Compute the following matrix expressions:

a) AB b) BA c) A^2

d) B^2 e) $(A+B)^2$ f) ABC

Problem 3.2 Give an example of a symmetric and a nonsymmetric 4×4 matrix.

Problem 3.3 Let A, B, and C be any $n \times n$ matrices. Simplify the following matrix expressions:

a) $AB(BC - CB) + (CA - AB)BC + CA(A - B)C$

b) $(A - B)(C - A) + (C - B)(A - C) + (C - A)^2$

Problem 3.4 We consider a linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 3 & 1 & 5 \\ 5 & -3 & 2 \\ 4 & -3 & -1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$$

a) Write out the linear system of equations.

b) Determine whether A is invertible, and find A^{-1} if it exists.

c) How many solutions does the linear system have?

Problem 3.5 Compute the matrix A^TA when A is the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & -1 & 6 & 5 \end{pmatrix}$$

Problem 3.6 Compute $|A|$ using cofactor expansion along the first column, and then along the third row. Compare both the results and the computations.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 1 & 0 & 8 \end{pmatrix}$$