FORK1003
Exercises for Lecture 1

August 3, 2015

1 Introduction to Linear Systems

1.1 Linear Equation

Exercise 1.1. Are these equations linear or nonlinear?

(a) $2x_1 + 2x_2 - 3x_3 = 9$

(b) $2x_1x_2x_3 = 0$

(c) $3x_1^2 - 3x_2 = 3$

(d) $x_1 - 2^{1/3}x_2 = 2$

(e) $x_1 - 2x_2^{1/3} = 2$

(f) $3(x_1 + x_2) - 2(x_3 - x_4) = 3$

(g) $(x_1 + x_2)(x_3 - x_4) = -5$

2 Solutions of Linear Systems

Exercise 2.1. Solve this linear systems by substitution:

(a) $$\begin{cases} x_1 + 2x_2 = 10 \\ -2x_1 + 3x_2 = 1 \end{cases}$$

(b) $$\begin{cases} -x_1 - x_2 = -2 \\ 5x_1 + 3x_2 = 5 \end{cases}$$

(c) $$\begin{cases} x_1 - x_2 + 3x_3 = 5 \\ 4x_2 - 3x_3 = -8 \\ -x_2 + 4x_3 = 2 \end{cases}$$

(d) $$\begin{cases} x_1 - 5x_2 - x_3 = 14 \\ 2x_1 - x_3 = 0 \\ -x_1 + 3x_2 = -10 \end{cases}$$

Exercise 2.2. How many solutions do these linear systems have?

(a)

$$\begin{cases} 16x_1 - 4x_2 = 8 \\ -2x_1 + \frac{1}{2}x_2 = -1 \end{cases}$$

(b)

$$\begin{cases} 3x_1 - 2x_2 = 4 \\ 9x_1 - 6x_2 = -2 \end{cases}$$

(c)

$$\begin{cases} x_1 - x_2 = 10 \\ x_1 + 3x_2 = 14 \end{cases}$$

3 Row Reduction

3.1 Coefficient & Augmented Matrix

Exercise 3.1. Write out the coefficient matrices of the following linear systems:

(a) $$\begin{cases} 3x_1 + 2x_2 = -3 \\ x_1 - x_2 + x_3 = 0 \\ -2x_1 - 3x_2 + 2x_3 = 4 \end{cases}$$

(b) $$\begin{cases} x_1 - x_3 = 2 \\ x_2 + 3x_3 = -1 \\ -4x_1 + 10x_2 - x_3 = 0 \\ x_1 + x_3 = 0 \end{cases}$$

(c) $$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 6 \\ x_2 - 10x_3 + 8x_4 - \frac{1}{2}x_5 = -2 \end{cases}$$

Exercise 3.2. Write out the augmented matrices of the following linear systems:

(a) $$\begin{cases} x_1 - 3x_2 + 8x_3 - x_4 = 1 \\ x_3 - 8x_4 = 13/3 \\ -2x_1 - x_2 + 3x_3 = 0 \end{cases}$$

(b) $$\begin{cases} 6x_1 = 8 \\ 3x_2 = -4 \\ -4x_3 = 2 \\ 18x_4 = 4 \end{cases}$$

(c) $$\begin{cases} 2x_1 - 7x_2 - 6x_3 - x_4 = 16 \\ x_2 + 11x_3 - \frac{3}{2}x_4 - \frac{1}{2}x_5 = 2 \end{cases}$$

Exercise 3.3. Express these augmented matrices as linear systems:

(a) $$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 1 & -2 & -3 & 6 \end{array}\right]$$

(b) $$\left[\begin{array}{cc|c} -2 & 0 & 10 \\ 13 & 2 & -16 \\ -3 & 4 & 0 \\ 4 & 2 & 3 \end{array}\right]$$

3.2 Elementary Row Operations

Exercise 3.4. Apply the given row operation to the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 6 \\ 0 & 3 & 5 & 7 \\ -2 & 4 & 0 & 13 \\ 16 & -6 & 7 & -1 \\ 0 & -1 & 0 & 2 \end{array}\right].$$

(a) $R3 \leftrightarrow R5$

(b) $R2 \to R2 - 3R1$

(c) $R4 \to -2R4$

(d) $R1 \to R1 + R2$

(e) $R5 \to \frac{1}{2}R5$

Exercise 3.5. Solve the following linear system using augmented matrices and elementary row operations:

$$\begin{cases} 2x_1 - x_2 = 4 \\ 3x_1 + 2x_2 = 13 \end{cases}$$

Exercise 3.6. Solve the following linear system using augmented matrices and elementary row operations:

$$\begin{cases} \phantom{2x_1 + {}} 3x_2 - 3x_3 = 9 \\ 2x_1 \phantom{{}+ 3x_2} - x_3 = -7 \\ 3x_1 + 2x_2 + x_3 = 0 \end{cases}$$

Exercise 3.7. Solve the general 2×2 linear system,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1, \\ a_{21}x_1 + a_{22}x_2 &= b_{.} \end{aligned}$$

What assumptions do you have to make about the coefficients?

3.3 Infinite or no Solutions

3.4 Echelon Forms

Exercise 3.8. Determine whether each matrix is in echelon form, and if so, whether it is in reduced echelon form.

(a) $\begin{bmatrix} 3 & 0 & 2 & 0 & 4 & 6 \\ 0 & -1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 2 & 4 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 4 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3.5 Pivot Positions & Basic Variables

Exercise 3.9. For each echelon form matrix, give the pivot positions, pivot columns, basic variables and free variables:

(a) $\left[\begin{array}{cccc|c} 1 & -2 & 0 & 6 & 2 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$