

# Leslie matrices

October 8, 2020

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[1]: # Python code: Leslie matrices
import numpy as np
import pandas as pd
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[2]: # Example: Rattus norvegicus
f = np.array([0,0,0.3964,1.4939,2.1777,2.5250,2.6282,2.6749,2.6018,
              2.4419,2.1865,1.9044,1.7259,1.4918,1.2415,0.9522,
              0.7141,0.4618,0.2518,0.0901,0.0035])
p = np.array([0.94697,0.99665,0.99926,0.99899,0.99863,0.99817,
              0.99753,0.99667,0.99553,0.99399,0.99196,0.98926,
              0.98572,0.98107,0.97511,0.96748,0.95797,0.94631,
              0.93247,0.91649])
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[3]: # Leslie matrix
n = f.shape[0]
def Leslie(f,p):
    matrix = np.zeros((n,n))
    matrix[0] = f
    for i in range(n-1):
        matrix[i+1,i] = p[i]
    return(matrix)
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[4]: A = Leslie(f,p)
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[5]: # dominant eigenvalue
eval, evec = np.linalg.eig(A)
max(abs(eval))
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[5]: 1.5624828630082712
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[6]: # Frobenius vector
def Frobenius(matrix):
    eval, evec = np.linalg.eig(matrix)
    dominant = max(abs(eval)).astype(np.complex)
    frob = evec[:,eval == dominant].real
    frob = frob/frob.sum()
    return(frob)
```

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[7]: np.reshape(Frobenius(A), (1,n))
```

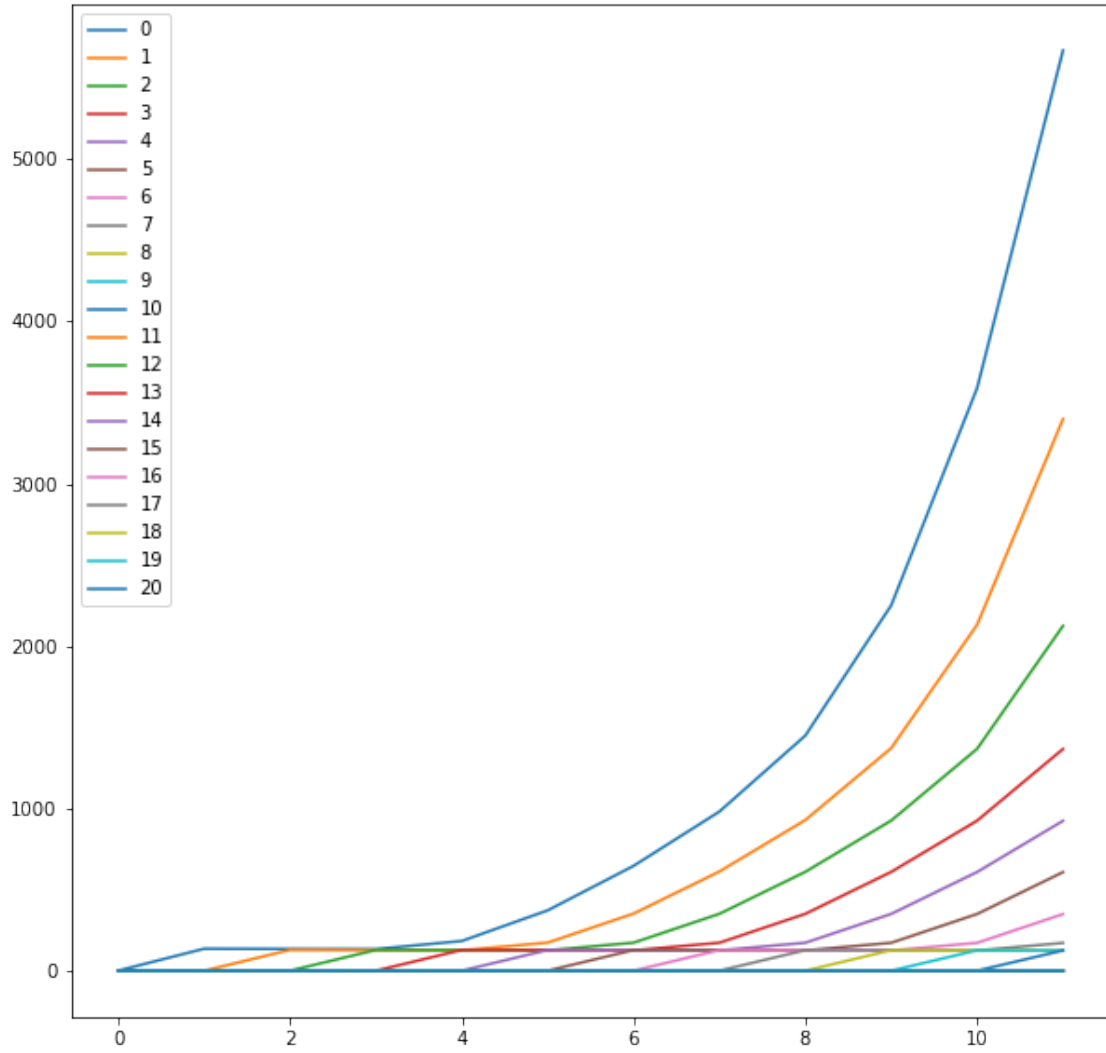
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[7]: array([[3.73628460e-01, 2.26444047e-01, 1.44440278e-01, 9.23743843e-02,  
          5.90605429e-02, 3.77473772e-02, 2.41143761e-02, 1.53952496e-02,  
          9.82025708e-03, 6.25693936e-03, 3.98041816e-03, 2.52701370e-03,  
          1.59993663e-03, 1.00934837e-03, 6.33761452e-04, 3.95516101e-04,  
          2.44901193e-04, 1.50150764e-04, 9.09380660e-05, 5.42706870e-05,  
          3.18330160e-05]])
```

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[8]: # initial vector: 5 in each age group  
initial = 5  
v = np.zeros((1,n)) + initial  
v = np.reshape(v, (n,1))
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[9]: # time series for age groups = rows of v  
months = 12  
rows = v.shape[0]  
for i in range(months-1):  
    last = v.shape[1]  
    w = np.reshape(v[:,last-1], (rows,1))  
    v = np.append(v, A.dot(w), axis=1)
```

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[10]: # transpose matrix to get time series as columns and plot them  
u = pd.DataFrame(np.transpose(v))  
u.plot(figsize=(10,10))
```

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[10]: <matplotlib.axes._subplots.AxesSubplot at 0x18695b198e0>
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