

Key Problems

Problem 1.

Write a Python program that defines a function `mx(vector)` that computes the maximal element in a one-dimensional ndarray `vector` of integers. The code should use recursion. Then use this code to find the maximal element in the following vectors:

a) `u = np.array([2,3,5,10,2])`

b) `v = np.array([18,53,59,22,93,19,89,66,24,28,73,83,32,5,12])`

Problem 2.

Write a Python program that defines a function `Gauss(matrix)` that returns an echelon form of the matrix `matrix`. Then use this code to find an echelon form of the following matrices:

a) `A = np.array([[1,1,1,3,-1],[1,2,4,7,3],[2,3,5,10,2]])`

b) `B = np.random.randn(10,5)`

Problem 3.

Find all complex roots of the following equations, and make a figure that shows the roots in the complex plane:

a) $x^4 + 16 = 0$

b) $x^4 + x^2 - 2 = 0$

c) $x^5 = 4 + 4i$

Problem 4.

Compute the rank of the following matrix with complex coefficients:

a)
$$\begin{pmatrix} 1 & i & 1 \\ i & 1 & i \\ 1 & -i & 1 \end{pmatrix}$$

b)
$$\begin{pmatrix} i & 2 & 2 \\ 2 & i & 2 \\ 2 & 2 & i \end{pmatrix}$$

c)
$$\begin{pmatrix} 1 & i & i & 1 \\ i & -1 & 1 & i \\ i & 1 & -1 & i \\ 1 & i & i & 1 \end{pmatrix}$$

Exercise problems

Problems from the textbook: [E] A.1 - A.6

Answers to Key Problems

Problem 3.

a) $\sqrt{2}(1+i), \sqrt{2}(1-i), -\sqrt{2}(1+i), -\sqrt{2}(1-i)$

b) $1, -1, \sqrt{2}i, -\sqrt{2}i$

c) $\sqrt{2}(\cos(\theta_i) + i\sin(\theta_i))$ for $i = 0, 1, 2, 3, 4$, with $\theta_i = 9^\circ + i \cdot 72^\circ$

Problem 4.

a) 2

b) 3

c) 3