Key Problems

Problem 1.
Check if the given sets are compact (closed and bounded), and if they are convex. It is useful to sketch the sets:

a) $D = \{(x,y) : x,y \geq 0 \text{ and } 2x + 3y \leq 6\}$

b) $D = \{(x,y) : 4x^2 + 9y^2 \leq 36\}$

c) $D = \{(x,y) : x,y \geq 1 \text{ and } 2x + 3y \geq 12\}$

d) $D = \{(x,y) : 4xy \leq 1 \text{ and } x,y > 0\}$

Problem 2.
Determine whether the functions are convex or concave:

a) $f(x,y,z) = x - y + z$

b) $f(x,y,z) = 1 - e^{x-y+z}$

c) $f(x,y,z,w) = (x + y + z + w)^4$

d) $f(x,y,z) = x^5 + y^5 + z^5$

Problem 3.
Solve the Lagrange problems:

a) $\max f(x,y,z) = x + 2y + 3z$ when $2x^2 + y^2 + 2z^2 = 9$

b) $\max / \min f(x,y,z) = x^4 + y^4 + z^4$ when $2x^2 + y^2 + 2z^2 = 9$

Problem 4.
Use the SOC to show that the given point is a solution of the Lagrange problem:

a) $(x^*,y^*) = (1,1)$ is a minimum for: $\min f(x,y) = x^2 + y^2$ when $xy = 1$

b) $(x^*,y^*,z^*) = (2,0,0)$ is a minimum for: $\min f(x,y,z) = x^2 + y^2 + z^2$ when $3x^2 + 2y^2 + 2z^2 = 12$

Problem 5.
Determine if there are any admissible points such that the NDCQ fails when the constraints are given by:

a) $xyz = 1$

b) $3x^2 + 3y^2 + 8z^2 = 1$

c) $x^3 + y^3 + z^3 = 0$

d) $xy - zw = 1$ and $x + y + z + w = 4$

Exercise Problems

Problems from the textbook:	[E] 6.1, 6.2, 6.3ab, 6.4, 6.11
Exam problems	Final exam 11/2019 Question 4

Answers to Key Problems

Problem 1.

a) Compact and convex set

b) Compact and convex set

c) Convex, but not compact set (not bounded)

d) Not convex and not compact set (not bounded)

Problem 2.

a) Convex and concave

b) Concave

c) Convex

d) Neither convex nor concave

Problem 3.

a) $f_{\text{max}} = 9$

b) $f_{\text{max}} = 81,\ f_{\text{min}} = 9$

Problem 5.

a) None

b) None

c) $(x,y,z) = (0,0,0)$

d) None