Key Problems

Problem 1.
We consider the quadratic form $f(x,y,z,w) = x^2 + ay^2 - 2ayz + az^2 + w^2 - 2xw$ with parameter a. Determine the definiteness of f when

a) $a > 0$ b) $a = 0$ c) $a < 0$

Problem 2.
Solve the unconstrained optimization problem $\max / \min f(\mathbf{x})$:

a) $f(x,y,z) = 5x^2 + 6xy + 2y^2 + 16xz + 10yz + 13z^2$
b) $f(x,y,z,w) = 2xz - 2yw - x^2 - y^2 - z^2 - w^2$
c) $f(x,y,z,w) = 2xy + 2xz + 2yw + 2zw$
d) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

Problem 3.
Find all stationary points of f, classify them as local maximum/minimum points or saddle points, and determine whether f has global maximum/minimum values:

a) $f(x,y,z) = xy + xz - yz$
b) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$
c) $f(x,y,z) = x^4 + y^4 + z^4 + z^2$
d) $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

Problem 4.
Determine whether f is a convex or concave function:

a) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$
b) $f(x,y,z) = e^{x-2y+z}$
c) $f(x,y,z) = x^4 + y^4 + z^4 + z^2$
d) $f(x,y,z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$
e) $f(x,y,z) = \dfrac{xy + xz + yz}{xyz}$ defined for $x,y,z > 0$

Problem 5.
Determine the range $V(f)$ of f:

a) $f(x,y,z) = \ln(1 + 2x^2 + 2xy + 3y^2 - 2xz + z^2)$
b) $f(x,y,z) = (x^2 + y^2 + z^2)e^{-x^2-y^2-z^2}$

Exercise Problems

Problems from the textbook:	[E] 5.1 - 5.14
Exam problems:	[Midterm 10/2017] Question 1-8
	[Midterm exam 10/2019] Question 3-7
	[Midterm exam 01/2020] Question 5-7

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Answers to Key Problems

Problem 1.

a. f is positive semidefinite when $a > 0$

b. f is positive semidefinite when $a = 0$

c. f is indefinite when $a < 0$

Problem 2.

a. $f_{\text{min}} = f(0{,}0{,}0) = 0$

b. $f_{\text{max}} = f(0{,}0{,}0{,}0) = 0$

c. f has no maximum or minimum

d. $f_{\text{min}} = f(0{,}0{,}0{,}0) = 0$

Problem 3.

a) Saddle point (0,0,0), no global max/min value

b) Local min (0,0,0,0), global min value $f_{\text{min}} = 0$, no global max value

c) Local min (0,0,0), global min value $f_{\text{min}} = 0$, no global max value

d) Local max (0,0,0), global max value $f_{\text{max}} = 16$, no global min value

Problem 4.

a) convex b) convex c) convex d) concave e) convex

Problem 5.

a) $V(f) = [0, \infty)$ b) $V(f) = [0, 1/e]$